Time-Reversal of \( \mu \text{s-long} \) Radiofrequency Signals Based on Approximate Temporal Imaging

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Abstract—We propose a purely analog way to time reverse optically-carried radiofrequency signals. Taking advantage of the time-space duality, we implement an approximate temporal imaging device that combines a time-lens and a single dispersive line. For military applications, this dispersive element must offer a group delay dispersion high enough to time-reverse \( \mu \text{s-long} \) signals. We propose to use spectral hole burning to tailor the absorption profile of erbium ions embedded in a crystalline lattice of yttrium orthosilicate. This way, we achieve a group delay dispersion 10\(^5\) larger than reached with km-long dispersive fibers, allowing us to extend the time-reversal process from the ns-range to the \( \mu \text{s-range} \).

I. INTRODUCTION

When a signal \( E(t) \) travels through an inhomogeneous medium, wavefront distortions occur and alter beam focalization, which prevents from targeting a source with accurate precision. Because of the time-reversal (TR) invariance of the propagation, a time-reversed signal \( E(-t) \) will follow the exact same path as the one taken in the first place, and will thus converge back to its source. This principle can be used in medicine to break up kidney stones with acoustics waves [1], as well as in telecommunications and electronic warfare devices with microwaves. For spectrally narrowband RF signals, a computer can achieve TR [2], but this method implies analog-to-digital conversion steps, too much time-consuming to be applied to broadband RADAR signals. A purely analog solution would then offer substantial processing time saving as well as signal bandwidth increasing. One way to achieve it relies on the up-conversion of the RF-signal to be time-reversed on an optical carrier at 1.53 \( \mu \text{m} \). 18 GHz bandwidth TR has thus been demonstrated by the way of an approximate temporal imaging (ATI) device [3]. However the highest group delay dispersion achievable with an optical fiber used as dispersive element only enables the processing of ns-long signals, too short for RADAR applications. By engraving a dispersive filter in the absorption profile of a rare-earth ion-doped crystal, we can implement an ATI scheme operating in the \( \mu \text{s-range} \).

II. TIME-REVERSAL IN THE FRAMEWORK OF TEMPORAL IMAGING

A. Time-space duality

In spatial optics, diffraction occurs during the propagation of an incident beam \( E(x, z = 0) \) along the \( z \)-axis. Provided the paraxial approximation is valid, the propagation over length \( z_j \) can be written as the convolution between the incident field of wavelength \( \lambda \) and a \( x \)-quadratic phase factor:

\[
E(x, z = z_j) \propto E(x, z = 0) \otimes \exp \left( -\frac{i\pi}{\lambda} \cdot \frac{x^2}{z_j} \right). \tag{1}
\]

We compare this equation with:

\[
E(t, z = z_j) \propto E(t, z = 0) \otimes \exp \left( -\frac{i}{2} \cdot \frac{t^2}{\beta''_{z_j}} \right) \tag{2}
\]

describing pulse propagation along the \( z \)-axis over length \( z_j \) in a dispersive medium \( j \) characterised by the second derivative \( \beta''_{z_j} \) of the propagation constant with respect to pulsation \( \omega \).

The mathematical similarity of diffraction and dispersion can be extended to the lensing effect. In the same way as a spatial lens induces a transmission coefficient \( \exp \left( \frac{2\pi i}{\lambda} \cdot \frac{x^2}{f^2} \right) \) where \( f \) stands for the focal length, we can define a temporal counterpart -called \emph{time lens}- that affects an incoming field as:

\[
E(t, z^+) = E(t, z^-) \cdot \exp \left( \frac{i}{2} \cdot r t^2 \right) \tag{3}
\]

where \( z^- \) (resp. \( z^+ \)) corresponds to position just before (resp. after) the time lens. The chirp rate \( r \) is defined as \( \Delta \omega / \Delta t \) [4].

B. Exact temporal imaging

By combining a time lens with two dispersive lines respectively located upstream and downstream from the lens, one can image a temporal object \( E(t) \), exactly as it is done in spatial optics (see Fig.1). This gives rise to the concept of temporal imaging [5].

The well-known thin lens equation allowing exact spatial imaging is now replaced by:

\[
\frac{1}{\beta''_{z_1}} + \frac{1}{\beta''_{z_2}} = r \tag{4}
\]
where subscripts 1 and 2 are attached to the two dispersive lines, and the magnification factor \( M \) is defined as:
\[
M = -\frac{\beta''_2}{\beta''_1}z_2 = 1 - r \cdot \beta''_2 z_2.
\] (5)
For TR, a \(-1\) magnification factor is required, leading to:
\[
\beta''_1 z_1 = \beta''_2 z_2 = 2/r.
\] (6)

C. Approximate temporal imaging

Although exact temporal imaging systems have already been demonstrated [6], the precise adjustment of the 3 parameters \( \beta''_1, r \) and \( \beta''_2 \) satisfying Eq. (4) remains a limiting issue.

With a simplified scheme combining only a time lens and a single dispersive line, the emerging field can be written as:
\[
\left\{ E(t) \cdot e^{\frac{i}{2} \pi r t^2} \right\} \otimes e^{-\frac{1}{2} \frac{r^2 t^2}{\pi^2 z_2^2}}.
\] (7)
Eq. (7) leads to an emerging wave proportional within a phase factor to \( E(t/M) \) [7] under the condition:
\[
\Delta \nu^2 \ll \frac{r}{\pi}
\] (8)
where \( \Delta \nu \) stands for \( E(t) \) frequency bandwidth.

One practical advantage of this approach is related to the time lens implementation. Indeed, instead of resorting to complex non-linear optical processes [6], one can achieve the quadratic phase modulation simply by transposing the waveform \( E(t) \) on a frequency chirped carrier, meaning no dispersive line between the object and the time lens. This simplified scheme is the spatial equivalent of a configuration where object and lens are placed side by side (Fig.2).

The price to pay for these advantages is that Eq. (8) establishes a constraint on the temporal resolution of an approximate temporal imaging device. Hence, \( \sqrt{\pi r} \) represents the duration of the shortest temporal substructure that can be imaged without distortion.

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**Fig. 2.** (a) Approximate spatial imaging: there is no first diffraction step and the object \( E(x) \) is placed next to the lens. We display the case \( M = -1 \), meaning \( z_1 = z_2 = 2f \) and leading to a spatially-reversed image \( \approx E(-x) \); (b) Approximate temporal imaging: there is no first dispersion step and the object \( E(t) \) is placed next to the lens. We display the case \( M = -1 \), meaning \( \beta''_1 z_1 = \beta''_2 z_2 = 2/r \) and leading to a time-reversed image \( \approx E(-t) \).

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### III. Time-Reversal Using a Rare Earth Ion-Doped Crystal

In reference [3], TR is performed in ATI configuration. A 18 GHz-bandwidth RF signal is first transposed on a chirped optical carrier (object and lens steps), then is sent through an optical fiber (dispersion step) and finally a time-reversed image of the input waveform emerges from the fiber. However, due to the limited group delay dispersion achieved even with a km-long dispersive line, this protocol can only handle signals in the ns-range.

When a signal travels through a random array of scatterers, each multiple reflection path creates a time-delayed echo of the incident wave. The duration \( \Delta t \) of the signal to be time-reversed, beginning with the ballistic wave and ending with the last measurable replica, depends of scatterers mean spacing. If we consider a scattering array of buildings in a town, the typical length scale between scatterers can be approximated to \( \approx 300 \) m. This corresponds to time-delays of few microseconds, and explains the necessity for TR protocols to reach the \( \mu s \)-range.

With this goal in mind, we consider ATI with components fundamentally different from the usual optical fibers. Relying on the strong dispersion that can occur in the vicinity of atomic resonance, we use a low-temperature rare-earth ion-doped crystal to provide us with the required dispersive power.

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### A. Properties of rare earth ion doped crystals

At low temperature, rare-earth ion-doped crystal (REIC) materials exhibit unusual properties of spectral sharpness [8]. In most solid-state materials, the optical resonance width of a single isolated ion, called homogeneous width \( \Gamma_h \), is considerably broadened when embedded in a solid lattice. In contrast, thanks to their specific electronic structure, rare earth ions are partially shielded from interactions with the environment, explaining their exceptionally narrow homogeneous linewidth. Furthermore, due to structural defects inside the crystalline matrix, each doping ion environment differs from
one crystal site to another. This results in a spreading of resonance frequencies, giving rise to the notion of inhomogeneous width $\Gamma_{inh}$ as illustrated in Fig. 3. Thus, the absorption profile is the sum of the contributions from different ion-assemblies, called spectral classes, that are resonant at the same frequency within the width $\Gamma_n$.

B. Spectral hole burning effect

The crystal must be prepared in order to operate as dispersive element in the ATI device. The idea is to tailor the absorption profile thanks to spectral hole burning (SHB).

As shown in Fig. 3, when a laser at frequency $\nu_L$ illuminates an absorption profile, some $\nu_L$-resonant atoms from the ground state $|g\rangle$ are promoted up to the excited state $|e\rangle$. As a result, a dip - called spectral hole - appears in the profile at $\nu_L$, with a shape determined by the laser spectrum.

C. Dispersive line shaping

Now we know that SHB can modify an absorption profile, we need to determine which shape to engrave in the REIC to build the ATI dispersive line.

In general, dispersion occurs because different spectral components travel with different velocities. In other words, the incident signal is affected by a frequency-dependent delay. Although this is an intrinsic property of optical fibers, no such thing exists in a REIC. We address this issue by implementing the desired frequency-dependent delay in the absorption profile.

We consider two consecutive pulses at frequency $\nu_L$, of width $\tau$ and separated by a duration $t_{12}$. If we illuminate the crystal with this two-pulse sequence, its associated spectrum - i.e a spectral grating of spacing $1/t_{12}$ and width $1/\tau$ (Fig. 4a) - will be engraved in the absorption profile over a $1/\tau$-broad and $\nu_L$-centered spectral interval (Fig. 4b).

Concerning laser monochromaticity, it should be noted that the engraving of a spectral shape only requires the laser linewidth to be negligible compared to the smallest spectral substructure of the shape to be engraved.

This spectral structure exhibits properties similar to the ones of a spatial grating. Indeed, a pulse incident on the $1/t_{12}$-spaced spectral grating undergoes $t_{12}$-delay in the same way as a beam is angularly deflected by a conventional grating.

It should be noted that this grating has been encoded over a $1/\tau$-interval around $\nu_L$, meaning that this $t_{12}$-delay will only affect the frequency components of the incident signal inside this spectral interval. As a result, by engraving several spectral gratings, each characterized by a specific spacing $1/t_{12}$ and a specific frequency $\nu_L^{(j)}$, we can implement the dispersive line in the REIC with a frequency-dependent delay $t_{12} = f(\nu_L^{(j)})$.

D. Complete TR-protocol

1) Dispersive line shaping : Instead of engraving multiple gratings step-by-step as described above, we can program one whose spacing linearly varies with the frequency (Fig. 5b). This is practically achieved by sending two consecutive chirped pulses with opposite chirp rate (see Fig. 5a). The locally sinusoidal character of the engraved aperiodic structure has been checked by transmission experiments shown in Fig. 5(i)-(iii).

2) Object and lens steps : The lens effect is provided by an optical carrier whose frequency is chirped with a rate $r$ (Fig. 6a). An up-conversion device is used to transpose the RF-signal to be time-reversed $E(t)$ (Fig. 6b) in the optical domain (Fig. 6c).

3) Dispersive step : Travelling through the aperiodic grating (Fig. 6d) engraved in step 1, the optically-carried signal undergoes dispersion, and an approximate time-reversed image emerges from the crystal (Fig. 6e). It should be noticed that the
carrier is time-reversed (ie negatively chirped) together with the signal.

4) $E(-t)$ detection: A photodiode detects only the envelope of the signal (Fig. 6f).

Fig. 6. Schematic description of our approach: (a) chirped optical carrier, (b) RF-signal $E(t)$ to be time-reversed, (c) optical carrier modulated by $E(t)$, (d) REIC engraved with a linearly varying spacing grating, (e) optical signal emerging from the REIC, (f) signal detected by an avalanche photodiode, proportional to $E(+t)$.

IV. EXPERIMENTAL RESULTS

A. Experimental setup

We use a 10 mm long 0.005%Er$^{3+}$:Y$_2$SiO$_5$ crystal cooled at 1.7 K in a liquid helium cryostat. The laser frequency is controlled by an electro-optic crystal inside the cavity [9]. A mode-hop-free tuning range $\Delta \nu_{\text{optic}}$ of 1.09 GHz in 6 $\mu$s can be reached ($r = 1.14 \cdot 10^{15}$ rad.s$^{-2}$). Values of $t_12$ range from 3 to 15 $\mu$s, leading to a smallest spectral substructure of 67 kHz, compatible with our laser linewidth ($\approx 10$ kHz). An acousto-optic modulator (AOM) is used to transpose the RF-signal $E(t)$ on the chirped optical carrier. The time-reversed output is finally detected by an avalanche photodiode placed after an AOM only opened during the echo pulse [10].

B. TR of a 6 $\mu$s-long signal

We demonstrate the TR of a 6 $\mu$s-long arbitrary waveform of Gaussian pulses (resolvable data number : 25, see Fig. 7a). A 50 signal-to-noise ratio has been measured, limited by the photodiode dynamics. The 1.6 $\%$ efficiency is consistent with the measured absorption profile modulation contrast (Fig. 5i-iii)). Thanks to the engraved REIC, we achieve a dispersive power $\beta''_m$ of 1.4 ms.nm$^{-1}$. For comparison purposes, that would be equivalent to the dispersive power of 10$^8$ km-long optical fiber ($\beta''_{\text{fiber}} = 17$ ps.nm$^{-1}.km^{-1}$).

Noticing that a sequence of $\tau_g$-long Gaussian pulses has a $1/\pi \tau_g$-broad spectrum, we can investigate the bandwidth issue. With our experimental parameters, Eq. (8) becomes:

$$\Delta \nu \ll \sqrt[\tau_g]{\frac{r}{\pi}} \approx 19 \text{ MHz}. \quad (9)$$

When increasing our signal input bandwidth, we observe that the time-reversed signal suffers from attenuation and distortion as shown in Fig. 7b). A -6dB bandwidth has been defined based on cross-correlation study between incident and normalized time-reversed signals, leading to a device bandwidth of 10 MHz.

V. CONCLUSION

We have demonstrated the time-reversal of $\mu$s-long optically carried RF signals with an approximate temporal imaging scheme, involving a time lens and a single dispersive line. Our approach originality lies in the non-persistent engraving of a REIC absorption profile thanks to spectral hole burning, allowing us to exploit the strong dispersion that can occur in the vicinity of atomic resonance. ATT device simplicity is counterbalanced by the bandwidth limitation given by Eq. (9). To increase this bandwidth and take advantage of the inhomogeneous broadening $\Gamma_{\text{inh}}$ offered by the REIC, one cannot dispense with exact temporal imaging, meaning a time lens and two dispersive lines. In our configuration, letting the signal go twice through the engraved crystal would lead to an even lower efficiency. A first step would then be to optimize the dispersive line engraving by enhancing the grating constraint, in a similar approach to the one successfully considered in the context of quantum memory for light [11].

REFERENCES