Broadband photonic arbitrary waveform generation using a frequency agile laser at 1.5 μm

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Received October 7, 2009; revised January 8, 2010; accepted January 11, 2010; posted January 11, 2010 (Doc. ID 118308); published February 22, 2010

We use a pulse-compression chirp-transform algorithm to generate broadband photonic arbitrary waveforms. A phase-locked frequency agile laser provides the needed broadband frequency scans. We theoretically show and experimentally demonstrate that the residual laser frequency errors do not significantly alter the waveform generation. The experiment exhibits features such as the operation at the telecom wavelength of 1.5 μm, the counterpropagating, crossed-polarized beam configuration, and the storage of the initial waveform in the active medium. The last specificity gives access to large time-bandwidth product values. © 2010 Optical Society of America

OCIS codes: 320.5520, 070.1170, 300.6240.

1. INTRODUCTION

Analog processing based on filtering by rare earth doped crystals (REIC) enjoys unrivaled capabilities in terms of bandwidth and time-to-bandwidth product. Several REIC-based architectures for the processing of optically carried broadband radar signals are today at the stage of transfer to industrial demonstrators. Meanwhile research for new architectures and materials is continuing. The chirp-transform algorithm is a fully coherent optical process which performs time-to-frequency Fourier transform in real time [1]. It was first considered for spectral analysis application [1–3] and can also be applied to wideband arbitrary waveforms generation (AWG) [4,5].

The latter application relies on the strong temporal compression of a much lower bandwidth initial waveform. The REIC-based filter operates in the optical spectral range. Hence the radio frequency (RF) signal to be processed has to be transposed on an optical carrier. The processing bandwidth Δ is limited by the inhomogeneous linewidth of the material absorption profile at the operating wavelength. This bandwidth may reach tens and even hundreds of gigahertz. In the chirp-transform algorithm, the optical carriers are frequency chirped over the bandwidth.

Broadband AWG with REIC technology was demonstrated in thulium doped yttrium aluminum at the wavelength of 793 nm in [5]. The optical carriers were produced by a frequency-stabilized monolaserchomonic laser. The frequency chirp was achieved by external electro-optic modulators (EOMs). Driving the EOMs over a large bandwidth was not easy. Only 20% of the available material bandwidth could be covered, in spite of sophisticated and expensive fast electronic means, such as ultra-fast pulse pattern generators. Anyway, an optical processing protocol is expected to rely on photonics means and is less convincing when depending too much on best electronics. Unlike an external EOM, a frequency agile laser can offer the straightforward, cheap, and effective full optical way for fast broadband scans [3].

In this paper we reexamine the conditions for performing broadband AWG with a chirped laser. We theoretically and experimentally show that the linewidth and chirp linearity requirements are less severe than mentioned in [4]. In addition we propose a variant of the architecture, offering a much larger time-bandwidth product (TBP). Finally, we perform the experimental demonstration in erbium doped yttrium orthosilicate (Er:YSO), at the wavelength of 1.5 μm, where advanced telecom equipment is available.

2. PHOTON ECHO CHIRP TRANSFORM

The chirp-transform algorithm [6] computes the Fourier transform 〈s(t)〉 of a signal s(t) according to

$$\tilde{s}(f) = |\tilde{s}(t) e^{-i\pi rt^2} \otimes e^{i\pi rt^2}|,$$

where ⊗ stands for the convolution product. The signal is first imparted a linear frequency sweep at rate r followed by a dispersion process at rate 1/r. This yields the signal Fourier transform, save for a phase factor. One actually obtains the temporal image of the signal spectrum with the time-to-frequency rule f = rt.

The chirp-transform algorithm has been extensively used for RF signal processing in compressive receivers using acousto-optic (AO) and surface acoustic wave (SAW) devices [7]. The algorithm can also be implemented with the help of stimulated photon echo (SPE) [1–3]. In the perturbation regime the SPE response can be expressed as the convolution product of the three driving fields according to

$$E_3(t) = E_1^*(-t) \otimes E_2(t) \otimes E_3(t).$$
As illustrated in [5], the main difficulty is the production of multi-gigahertz range, as required to process fast signals. The dispersion filter is chirped at rate $-r_3$ over the same interval. The delay of the first two excitations is a function of frequency that varies from $T=(r_1-r_2)/r_3$ at $v_1$ to $T+\Delta/r_3$ at $v_1+r_3$. The characteristic time evolution of the material response is given by the inverse spectral span $\Delta^{1/2}$. The figure illustrates the response to a rectangular probe of duration $\Delta/r_3$. With respect to the time-varying fields, the response undergoes a $\Delta^{1/2}/r_3$ compression.

When fields $E_1(t)$ and $E_2(t)$ are chirped at rates $r_1$ and $r_2$, respectively (see Fig. 1), their product $E_1(-t) \otimes E_2(t)$ operates as a dispersive (or compressive) filter on the probe $E_3(t)$, making this field undergo dispersion at rate $1/r_3=1/r_2-1/r_1$. If $E_2(t)$ is chirped at rate $-r_3$ and modulated by a RF signal $s(t)$, the echo power displays the power Fourier transform of $s(t)$ in accordance with Eq. (1).

The compression effect simply reflects the fact that the spectrum of a long monochromatic pulse is a narrow feature. Displaying this feature in the time domain may result in a short pulse. Alternatively, multiplying $s(t)$ by a chirped field can be regarded as a way to spread the spectrum of the signal, making this spectrum cover the entire $\Delta$ interval swept by the chirped carrier. The response in the time domain can be made as short as $\Delta^{-1}$, which is $\Delta^{2}/r_3$ times shorter than the $\Delta/r_3$-long probe field. The probe pulse and the material response must be contained altogether within the atomic coherence lifetime, which limits the compression effect.

Many REIC offer an inhomogeneous broadening in the multi-gigahertz range, as required to process fast signals. As illustrated in [5], the main difficulty is the production of fast, broadband, phase-coherent optical chirps. In previous works [1-3], we programmed the dispersive filter over $\Delta$ with the help of two successive pulses $E_1(t)$ and $E_2(t)$ with opposite chirp rates $-2r$ and $2r$. The signal carrier $E_3(t)$ was chirped at rate $-r$. We directly scanned a frequency agile laser to generate the desired chirp rates. We have developed a specific self-heterodyning servo-loop to improve the linearity and spectral purity of the sweeps [8]. Indeed, the frequency error on the optical chirp must be kept less than the inverse duration of the pulses. The pulse duration, $\Delta/(2r)$ or $\Delta/r$, falls in the microsecond range, to be consistent with the atomic coherence lifetime, and the locked laser satisfies the condition on the frequency error quite easily.

One may question the use of microsecond pulses for the typical 10 ms population lifetime of the REIC bottleneck state. Programming pulses in the millisecond range can engrave the filter more efficiently at lower intensity. Temporally overlapped pulses can achieve such a long storage. Indeed, with $r_1$ close to $r_3$, one can simultaneously stretch the engraving step and satisfy the short probe pulse condition, with $r_3 \gg r_1, r_2$. However, millisecond pulses seem to impose a very demanding condition on laser stability. With frequency errors smaller than the inverse duration of the pulses, laser control in the sub-kilohertz range would be required. We shall address this issue in the next section.

3. IMPACT OF LASER FREQUENCY ERRORS

As noticed above, it has been considered that the laser frequency errors should be kept smaller than the inverse pulse duration. With 1 ms-long programming pulses, a sub-kilohertz frequency precision would be required. Maintaining such small frequency errors is out of reach of available frequency agile lasers. However, to derive this very strict condition, one implicitly assumes that the different fields are uncorrelated. This is not necessarily true when $E_1(t)$ and $E_2(t)$ are overlapped with $r_1 \equiv r_2$. Indeed the two fields can be simultaneously derived from a single
source, the chirp difference $r_1 - r_2$ being provided by, e.g., an AO modulator. Then the critical parameter is no longer the inverse field duration.

To be specific, let the frequency agile laser scan begin at time $t=0$. The AO shifter makes the instantaneous frequencies of fields $E_1(t)$ and $E_2(t)$, respectively, evolve as $v_1(t) = v_1 + r_1 t$ and $v_2(t) = v_2 + r_2 t$. During engraving, an atom at frequency $v$ successively interacts with the two fields at times $(v - v_1)/r_1$ and $(v - v_2)/r_2$. The interaction with the field depends on the frequency errors that accumulate during the delay $v/r_2 + v_1/r_1 - v_2/r_2$. This time interval varies from $T = (v_1 - v_2)/r_2$ to $T + \Delta/r_2$ as the laser is scanned from $v_1$ to $v_1 + \Delta$ and can remain much smaller than the duration of the engraving fields.

Our chirped laser is self-stabilized with an optical phase lock loop (OPLL) [8]. Most of the $1/f$ technical noise is suppressed. We have observed that the surviving frequency fluctuations exhibit a white noise spectrum between 0 and 1 MHz [9]. The average response intensity can be calculated easily within the frame of this simple frequency white noise model. The calculation is detailed in the Appendix. We assume that none of the participating fields carries any additional signal $s(t)$. In other words, each one of $E_1(t)$, $E_2(t)$, and $E_3(t)$ reduces to a single, monotonously chirped monochromatic field. In these conditions, the material response reduces to a single brief emission around time $t = t_3 + T$. As a function of the quiescent laser linewidth $\Gamma/(2\pi)$, the average temporal profile of this short pulse reads as

$$I_s(\tau, \Gamma) = I_s(0,0) e^{-\gamma T} \frac{2}{\gamma^2 + (2\pi\Delta)^2} \left[ 1 - e^{-\gamma} \cos(2\pi\Delta \tau) - e^{-\gamma} \sin(2\pi\Delta \tau) \right],$$

where $\tau = t - t_3 - T$, and $\gamma = \Gamma/r_3$.

In the absence of frequency errors, when $\Gamma = 0$, $I_s(\tau, \Gamma)/I_s(0,0)$ reduces to $\left[ \sin(\pi\Delta \tau)/(\pi\Delta \tau) \right]^2$, which corresponds to the squared Fourier transform of a rectangular pulse [see Fig. 3(b)]. The $\tau_{1/2}$ full width at half maximum equals $0.89\Delta^{-1}$.

As $\Gamma$ is increased, the frequency errors affect the response in two different ways. First the temporal profile $I_s(\tau, \Gamma)$ is stretched as $\Gamma$ gets larger than the inverse probe pulse duration. This reflects the loss of coherence between atoms with different transition frequencies. One might expect that $\tau_{1/2}$ tends to the inverse width of a coherently scanned interval given by $\Gamma/r_3$. On the contrary, $\tau_{1/2}$ varies much more slowly, as illustrated in Fig. 3. In other words, the response coherently combines contributions from a spectral interval much broader than $r_3/\Gamma$. The frequency errors are more significantly reflected in the decay factor $e^{-\gamma T}$ that affects the response intensity as a whole. Finally, the pulse compressor should work properly provided $\Gamma T \ll 1$. In the following we experimentally explore the chirped laser capabilities and limitations in this pulse-compression context.

4. EXPERIMENTAL

A. Pulse-Compression Setup

The pulse-compression experiment takes place in 0.005% at. Er$^{3+}$:YSO at 1.5 μm. At this wavelength a lot of optical telecom devices are available such as erbium-doped fiber amplifiers, pig-tailed modulators, couplers, low-cost fibers, etc. The setup is depicted in Fig. 4. The laser beam is split in two arms that counterpropagate to the crystal. The fields $E_1(t)$ and $E_3(t)$ follow the same path in the opposite direction to the other engraving field $E_2(t)$. The signal $s(t)$ is carried by $E_2(t)$. As noticed in Section 2, this is less demanding in terms of modulation bandwidth than transposing the signal on $E_2(t)$. With the counterpropa-
Fig. 4. (Color online) Experimental setup. The chirped laser beam is split in two arms that counterpropagate to the Er\(^{3+}\) : YSO. On one arm, the AO modulator AO2 transposes the signal on \(E_2(t)\). On the other arm, the shifter AO1 takes care of \(E_1(t)\) and \(E_2(t)\) time ordering and provides the chirp rate difference \(r_1−r_2\). \(E_1(t)\) and \(E_2(t)\) are cross-polarized with \(E_3(t)\). The SPE response is detected on photodiode PD.

The \(r_1−r_2\) chirp rate is obtained as the laser is driven back to initial frequency in 15 \(\mu\)s. This corresponds to \(r_3\) = \(10^{10}\) Hz/s. The OPLL does not react instantaneously to the chirp rate change. To account for this latency time, we reduce the working spectral interval to \(\approx 1.2\) GHz. This imposes a waiting time of \(\approx 300\) \(\mu\)s between the engraving and the probing steps. The AO device AO1 is opened to let \(E_3(t)\) probe the engraved spectral interval.

Both AOs offer a bandwidth of \(\approx 35\) MHz centered at 80 MHz. On the path followed by \(E_2(t)\), one uses the 17 MHz-wide lower side of the AO2 bandwidth to transpose \(s(t)\) on \(E_2(t)\) (see Fig. 5). The upper side of the AO1 bandwidth is used to generate the small \(r_1−r_2\) chirp rate difference that is needed to match \(r_3\). This represents a frequency variation of 18 MHz over the explored spectral interval, which indeed matches the upper side of the AO1 bandwidth.

The maximum initial frequency shift of \(E_1(t)\) and \(E_2(t)\) amounts to 17 MHz. This corresponds to an initial delay \(T=11\) \(\mu\)s, consistent with the optical coherence lifetime \(T_2=150\) \(\mu\)s.

The experiment is controlled by an arbitrary waveform generator [(AWG Tektronix 5004). This device drives AO1, feeds the signal \(s(t)\) to AO2, and controls the laser chirp. The arbitrary waveform generator delivers a voltage ramp to the intra-cavity EOM through a high voltage amplifier and simultaneously acts as a matched local oscillator to the OPLL [8].

**B. Results**

As a demonstration of the processing capabilities, we generate a sequence of ten short pulses by feeding ten evenly spectrally spaced monochromatic waveforms to \(E_2(t)\) through AO2. All the waveforms are applied during 0.8 ms, while the laser is scanned over 1.2 GHz. The resulting spectral grating is engraved over the erbium ion absorption profile. With a full width at half maximum of 0.68 GHz, the ion spectral distribution is not uniform over the 1.2 GHz-wide laser scanning range. By optimizing the SPE intensity, we can center the absorption profile within the scanned interval, as illustrated in Fig. 6. Each monochromatic waveform gives rise to a pulse of about 1 ns duration. The temporal profile of each pulse should be given by the power Fourier transform of the absorption profile over the scanned interval.

In a first experiment, the ten monochromatic waveforms at the input of AO2 are spread over an interval of 1.54 MHz. This gives rise to a 1 \(\mu\)s-long burst of ten pulses (see Fig. 7). The pulse duration is close to 1 ns as expected, which represents a 10\(^6\) compression with respect to the input waveform. To observe the decay as a function of \(T\), we increase the spectral spacing of the monochromatic waveforms. As shown in Fig. 8, the burst is stretched to 10 \(\mu\)s, with the same pulse duration. This corresponds to an available TBP of \(\approx 10^4\), which represents \(\approx 1/10\) of the material maximum capacity in the present magnetic field conditions. The exponential decay during the burst partly reflects the atomic coherence relaxation at \(6.7 \times 10^3\) s\(^{-1}\). The data fit in Fig. 8 leads to
a laser frequency error rate of at most $1.1 \times 10^5 \text{ s}^{-1}$, which corresponds to a quiescent laser linewidth of $\approx 20 \text{ kHz}$. According to Eq. (3), the SPE intensity depends on $\gamma = \Gamma \lambda / r_3$ expressing the fraction of $\Delta$ that is spanned by the probe during the laser coherence time. With the measured $\Gamma$ value, one gets $\gamma \approx 1.1$. As can be seen in Fig. 3, with such a $\gamma$-value, the pulse width is not affected by the laser frequency errors. Hence, despite the linewidth being much larger than the inverse engraving pulse duration, the processing capabilities are not significantly affected, except for faster decay, and therefore smaller signal-to-noise ratio on long time scales.

In [8] we performed a self-heterodyning measurement of the free running laser coherence time. This time reaches $180 \mu s$ at 5 mW, which corresponds to a linewidth of 2 kHz. From the analysis of the servo error signal, we also estimated the locked-chirp laser frequency error to $\approx 65 \text{ kHz}$, on a time scale of a few tens of microseconds, with a chirp rate of $6 \times 10^{14} \text{ Hz/s}$. In the present experiment, the 20 kHz quiescent laser linewidth, as deduced from the SPE response decay, appears to be intermediate between these two linewidths. This point should be clarified.

5. CONCLUSION
A locked-chirp frequency agile laser can be stable enough to deliver the millisecond-long engraving pulses for broadband waveform generation using the chirp-transform algorithm. This important result emphasizes the possibility to resort to simple photonic means to synthesize a complex wideband RF signal. We show theoretically and demonstrate experimentally that the laser frequency errors do not significantly alter the signal compression procedure. A $10^3$ time-bandwidth product (TBP) has been obtained experimentally. Experiments are conducted at the telecom wavelength in Er$^{3+}$:YSO. However this material offers a limited bandwidth. To reach a bandwidth of several tens of gigahertz, alternative materials, such as Er$^{3+}$:LiNbO$_3$, should be investigated.

Note: The research reported in this paper was performed under the smart and enthusiastic guidance of Ivan Lourgeré. Ivan passed away off Brazil on June 1, 2009, in the crash of the AF447 flight, on his way back from a scientific meeting with Brazilian colleagues.

APPENDIX A: SPE RESPONSE WITH LASER FREQUENCY ERRORS
According to Eq. (2), the statistical average of the SPE response intensity reads as

$$I_e = \int_{\nu_1}^{\nu_1 + \Delta} d\nu \int_{\nu_2}^{\nu_2 + \Delta} d\nu' \langle \tilde{E}_1(\nu) \tilde{E}_2^*(\nu') \tilde{E}_2(\nu') \rangle$$

$$\times \langle \tilde{E}_2(\nu') \tilde{E}_2^*(\nu') \rangle e^{2\pi i (\nu - \nu') t}, \quad (A1)$$

where the probe pulse is assumed to be statistically independent of the engraving pulses. The sum runs over the
spectral interval \( \Delta \), where the excitation is uniformly distributed. The frequency errors are assumed to be described by a stochastic phase \( \phi(t) \). An external modulator is used to shift \( E_1(t) \) with respect to \( E_2(t) \). This operation does not affect the stochastic phase. Therefore the fields can be described as

\[
E_j(t) = E_0 e^{2\pi i \omega_j t + \pi \epsilon_j j^2 + i \phi(t)},
\]

where \( \epsilon_1 = 1, \epsilon_2 = -1, \) and \( 1/\Gamma = 1/r_2 - 1/r_1 \). In the frame of the white frequency noise model, the statistical average of a two-time phase factor can be written as

\[
\langle e^{i \phi(t) - i \phi(t')} \rangle = e^{-1/2(\Gamma')^2 t - t'}. \tag{A3}\n\]

According to this model, \( \tilde{E}_1(v)\tilde{E}_2(v) \) is decorrelated from \( \tilde{E}_2'(v')\tilde{E}_1'(v') \) in Eq. (A1), as soon as \( |v - v'| \) is larger than the frequency interval that is spanned by the chirped laser during the coherence time \( \Gamma^{-1} \). Therefore, under the assumption that \( \Delta \gg r_2 \Gamma^{-1} \), the four-field average \( \langle \tilde{E}_1(v)\tilde{E}_2(v)\tilde{E}_2'(v')\tilde{E}_1'(v') \rangle \) reduces to \( \langle \tilde{E}_1(v)\tilde{E}_2(v) \rangle \times \langle \tilde{E}_2'(v')\tilde{E}_1'(v') \rangle \) over most of the integration domain. We perform the calculation under this assumption and will check its validity in the present experiment.

According to Eqs. (A2) and (A3), the two-field average reads as

\[
\langle \tilde{E}_1(v)\tilde{E}_2(v) \rangle = E_{01} E_{20} \int dt \int dt' \times e^{2\pi i [((v-1)/r_1) t - (v-2)/r_1 t']} [-i \pi (r_2^2 - r_1^2 t^2) - 1/2(\Gamma')^2 t - t']. \tag{A4}\n\]

With the change in variables,

\[
t, t' \rightarrow u = t - t', \quad v = \frac{r_1 t - r_2 t'}{r_1 - r_2}, \n\]

the expression reduces to

\[
\langle \tilde{E}_1(v)\tilde{E}_2(v) \rangle = \eta_{\sqrt{r_2}} E_{01} E_{20} e^{i (\pi v_2 - \pi/2) T / r_2} \int df e^{-i (\pi f_3) (v - v_1) f + f^2 T^2} \mathcal{L}(f), \tag{A5}\n\]

where \( \eta = 1/\sqrt{r_2(r_2 - r_1)}, \) \( T = (v_1 - v_2)/r_2, \) and

\[
\mathcal{L}(f) = \frac{\Gamma}{(2\pi f)^2 + (\Gamma f)^2}. \n\]

In quite the same way one obtains the spectral autocorrelation function of the probe field:

\[
\langle \tilde{E}_2'(v')\tilde{E}_1'(v) \rangle = r_2^{-1} E_{20}^2 \int df' e^{i (\pi f_3) (v - v_2 + f')^2 - (v' - v_2 + f')^2} \mathcal{L}(f'). \tag{A6}\n\]

With the help of Eqs. (A5) and (A6), one can rearrange the six-field product as

\[
\langle \tilde{E}_1(v)\tilde{E}_2(v)\tilde{E}_1'(v')\tilde{E}_2'(v') \rangle \langle \tilde{E}_2'(v')\tilde{E}_1'(v) \rangle \langle \tilde{E}_1'(v)\tilde{E}_2(v) \rangle \tag{A7}\n\]

\[
= \eta^2 |E_{10}|^2 |E_{20}|^2 |E_{20}|^2 e^{2i \pi (v - v')/T_1 + T} \times \int df \int df' \mathcal{L}(f) \mathcal{L}(f') \times e^{i (\pi f_3) (v - v_2) f + f^2 T^2} e^{-2i \pi (v - r_1 + r_2) T} \mathcal{L}(f'), \n\]

where \( T_1 = \frac{(v_2 - v_1)}{r_3}. \) Assuming \( T_1^2 \ll T_3 \), we can neglect the quadratic phases. Hence we are simply left with the following product of Lorentzian Fourier transforms:

\[
\langle \tilde{E}_1(v)\tilde{E}_2(v)\tilde{E}_1'(v')\tilde{E}_2'(v') \rangle \langle \tilde{E}_2'(v')\tilde{E}_1'(v) \rangle \langle \tilde{E}_1'(v)\tilde{E}_2(v) \rangle \tag{A8}\n\]

Substitution of Eq. (A8) into Eq. (A1) leads to

\[
I_e = \eta^2 |E_{10}|^2 |E_{20}|^2 |E_{20}|^2 \int \frac{dt}{\sqrt{r_2}} \int \frac{dt'}{\sqrt{r_2}} \int \frac{dv}{\sqrt{T}} \int \frac{dv'}{\sqrt{T}} \times e^{2i \pi (v - v') r_3} e^{-v' - v'} (\Gamma / 2r_3)^2, \tag{A9}\n\]

where \( T = t - t_3 - T \). Integration is easily performed. Expressing the average response intensity as a function of \( \tau \) and \( \Gamma \), one finally obtains

\[
I_e(\tau, \Gamma) = I_e(0, 0) e^{-\tau T} \frac{2}{\gamma^2 + (2\pi \Delta)^2} \left[ 1 - e^{-\gamma} \cos(2\pi \Delta \tau) - \gamma e^{-\gamma} \sin(2\pi \Delta \tau) / 2\pi \Delta \right]. \tag{A10}\n\]

where \( \gamma = \Gamma \Delta / r_3 \).

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