Wideband radio frequency spectrum analyzer: improved design and experimental results

V. Lavielle, F. De Seze, I. Lorgeré, J.-L. Le Gouët*

Laboratoire Aimé Cotton, CNRS, Bâtiment 505, Campus Universitaire, 91405 Orsay, France

Abstract

We report on significant progress accomplished in spectral-hole-burning-based instantaneous spectral analysis of optically carried broadband radio frequency signals. This concept aims at becoming the next generation spectrum analyzer for broadband radar and astronomical applications. Since the initial demonstration (Opt. Lett. 26 (2001) 1245) the spectral range has been broadened by two orders of magnitude and the channel number has been improved by nearly one order of magnitude. We discuss the next steps to be considered, namely 100% duty cycle and 1000-channel operation. On the way to 100% duty cycle operation, we demonstrate frequency offset-locking of a frequency-chirped laser to a fixed frequency laser. We examine two-dimensional angular addressing as a technique to increase the spectral channel number.

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1. Introduction

Astronomers are expecting valuable information from (sub)millimeter spectroscopy of molecular and atomic species that are present in dusty regions of space [1]. In heterodyne receivers for (sub)millimeter astronomy, the input signal is first processed by the low noise “front-end” of the system, that includes mixing and amplification stages, before being conveyed to the spectrometers.

The most widely used spectrometers in radio astronomy are the acousto-optic spectrometer (AOS) [2–4], the digital autocorrelation spectrometer (ACS) [5] and the filterbank spectrometer (FBS). The spectrometer bandwidth ranges from 0.2–1 GHz for digital ACS to 1–1.5 GHz for the AOS. Spectrometers based on all types of technology, except the digital autocorrelation type, must be designed with fixed bandwidth and resolution, i.e. they are inflexible for different kinds of observations. The digital ACS is in this regard flexible, but intolerable power consumption at high frequency limits its processing capabilities in terms of bandwidth. To be able to process very wide bandwidth signals (1–4 GHz), one resorts to
a pre-filtering intermediate frequency (IF) stage that divides the input signal broad band into several narrower sub-bands [1].

In order to cope with increasing bandwidth demand, one needs alternative technology with great potential for future space laboratories. The spectral hole burning (SHB) technology has this potential. Specifically, the inhomogeneously broadened absorption bands of rare-earth-doped crystals have bandwidths as large as 200 GHz, together with spectral resolution below 10 kHz at low temperature [6]. These materials are particularly suited to processing broadband radio frequency (RF) signals with impressive time-bandwidth product. Various optical functions, such as time-domain correlation [7,8], real-time data routing [9,10] and true-time delay generation for radar application [11,12], have been demonstrated recently that illustrate SHB signal processing potential. We recently proposed an opto-electronic architecture to exploit the large bandwidth and high resolution of SHB for spectral analysis of RF signals [13–15]. Ultimately, the SHB spectrometer would feature bandwidth in excess of 10 GHz, with flexibility, resolution better than 1 MHz and low consumption.

The price to pay for these attractive properties is the cooling to an operating temperature of about 5 K. However, low-temperature operation should not be regarded as a major inconvenience. The launching of ISO satellite, with a 2000 l tank of superfluid helium, recently demonstrated that large-scale extreme cryogenic conditions could be achieved in embarked experiments. This is confirmed by the forthcoming Planck and Herschel missions, where some devices are cooled down to 0.1 K [1].

We first describe the basics of the SHB spectrum analyzer in Section 2. Then we present our new experimental results in Section 3, highlighting some important technical aspects. In Section 4, we report on the frequency offset-locking demonstration of a chirped laser to a fixed frequency reference, which opens the way to 100% duty cycle operation of the SHB spectrum analyzer. Section 5 is devoted to an investigation of two-dimensional angular addressing as a mean to reach 1000-channel resolution. Finally we conclude in Section 6.

2. SHB spectrometer

Owing to the broad bandwidth capabilities of opto-electronic components, it is worthwhile to process RF signals in the optical domain, after transfer on an optical carrier. This is the way the SHB spectrometer operates. The concept relies on the engraving of monochromatic gratings in an SHB material. Each grating is able to diffract a single spectral component, with a resolution ultimately determined by the homogeneous line width of the SHB medium, which is usually less than 1 MHz at the temperature of 5 K. A large number of gratings can coexist within the inhomogeneous width of the absorption line, which may reach tens of GHz. By varying the laser frequency in synchrony with the angle of incidence during the engraving procedure, one associates a specific diffraction angle with each specific spectral component. Therefore, the different spectral components of an incident polychromatic probe beam are diffracted and simultaneously retrieved in different directions. The stack of monochromatic gratings works as a spectrometer which is expected to exhibit a resolution of less than 1 MHz and a bandwidth of several tens of GHz. As depicted in Fig. 1, RF spectral analysis can be performed after
transfer of the investigated microwave signal on an optical carrier with the help of a Mach–Zehnder electro-optic modulator (MZM). The bandwidth of the SHB medium indeed matches that of integrated MZM developed for high flow telecommunication. To be more specific, SHB media can be devised to cover the bandwidth in excess of 50 GHz offered by fast integrated MZM [6]. Since the spectrometer relies on the angular separation of the frequency components, the number of frequency channels equals the number of different angular directions that can be addressed by the device. For a given setup, this number is fixed. However, the engraving laser frequency scanning range, which determines the spectrometer bandwidth, can be varied easily. By reducing this range while the channel number is kept fixed, one is able to zoom on a specific spectral region with improved spectral resolution.

This design is reminiscent of the well-known AOS. In the latter device a Bragg cell achieves two functions. On the one hand, it transfers the RF signal on the optical carrier. On the other hand, it accomplishes the angular separation of the optically carried spectral components. The acoustic wave absorption limits the bandwidth to $B \approx 1 \text{GHz}$. The integrated MZM offer a much larger bandwidth. However, an MZM only transfers the RF signal on the optical carrier but does not achieve the angular separation of the spectral components, since the carrier and the side bands propagate along the same direction. In the SHB spectrometer, the SHB crystal is intended to complement the MZM component by achieving the missing angular separation.

The beam configuration is strongly constrained by the material limitations. Owing to the short lifetime of the engraving, the gratings must be refreshed continuously and simultaneously diffract the impinging signal beam. This condition is satisfied in the “box configuration” presented in Fig. 1. The probe beam that carries the RF signal to be processed propagates along $\vec{k}_3$. It lies out of the plane defined by the engraving beam wave vectors $\vec{k}_1$ and $\vec{k}_2$. This non-coplanar arrangement is consistent with simultaneous writing and readout since the diffracted beam is directed along $\vec{k}_3 + \vec{k}_1 - \vec{k}_2$ which differs from all the incident wave vectors. The wave vectors $\vec{k}_2$ and $\vec{k}_3$ are headed in fixed directions while $\vec{k}_1$ rotates in synchrony with the frequency scan of the engraving beams. Therefore, the different spectral components that are carried by the probe beam are diffracted in different directions.

3. Demonstration of 100-channel operation

3.1. Experimental setup

In the engraving stage, the angular scan is achieved by a frequency-shift-compensated pair of acousto-optic deflectors that are crossed successively by beam #1. The deflectors, respectively, diffract the light beam in order $-1$ and $+1$ so that the respective frequency downshift and upshift are subtracted to each other. The two deflectors are oriented in perpendicular directions and are slanted at $45°$ from horizontal. When the two deflectors are driven in synchrony with a fixed detuning, the emerging beam is scanned horizontally (see Fig. 2). To increase the channel number with respect to our initial experiments, we use a larger aperture deflector. This device (A&A DTS XY 250) offers a 5 mm clear aperture, and a 69 mrad scanning range as the common driving frequency is varied from 85 to 125 MHz.

An optical relay, with a magnification factor $m_x = 0.25$, images the deflector on the SHB crystal so that the illuminated spot does not move as the beam direction is varied.

The two output channels of a 1 Gigasample/s arbitrary wave form generator (Sony/Tektronix AWG520) separately drive the two AOs that are combined in the deflector. This enables us to set a fixed frequency detuning between the two chirped synchronized driving waves. The acoustic wave detuning leads to a fixed frequency shift of the deflected beam with respect to the fixed direction engraving beam. Since the laser is simultaneously frequency chirped, each ion interacts with the two beams at two successive moments. As discussed in Ref. [14], the interaction delay makes the setup operate in photon echo configuration, which optimizes the spectral resolution and the signal diffraction efficiency.
The synchronized laser frequency chirp and beam angular scan must be repeated over the spectrometer bandwidth with precision better than the desired resolution, since the grating storage is accumulated in the active crystal for several engraving cycles. To satisfy the repeatability requirements of our application, it was necessary to develop a novel electro-optically tuned extended-cavity diode-laser [16,17]. The item we are using in this experiment, equipped with an improved AR-coated chip, is easily scanned without mode hopping over a 3.5 GHz interval. Careful adjustment should permit to increase the scanning range to 10 GHz.

In previously studied photon echo applications, accumulation is regarded as a demanding procedure [18,19]. In these applications, the storage coordinate is the time delay of the engraving fields. The delay-dependent phase shift that is stored in the material is affected by laser frequency fluctuations the more as one increases the delay range. A 1 μs delay storage range requires at least 100 kHz laser stability during a 10 ms long accumulation process [19]. The present situation is somewhat different. All the gratings associated with the different spectral channels are engraved with the same minimized delay. For 33 MHz resolution, this delay does not need to exceed ~10 ns (see the appendix, Eq. (A.17)), which is much smaller than the inverse jitter range of a free running extended cavity diode laser over a 10 ms time interval. However, we shall see in Section 5 that the 2D operation of the spectrum analyzer might demand excellent laser stability, irrespective of the spectral resolution to be achieved.

Since we split the three needed beams from a single laser, we have to operate in a sequenced mode, where grating engraving and signal diffraction are alternate. Engraving takes place during the first half repetition period, while the laser frequency is kept fixed for readout during the second half period. With a repetition rate of 2 kHz, the 250 μs-long writing step is followed by a same duration step devoted to RF signal spectral analysis. Since the shelving state lifetime is ~10 ms, engraving is accumulated over ~20 writing steps.

The modulated probe beam to be spectrally analyzed is directed to the Tm$^{3+}$:YAG crystal and is diffracted on the engraved gratings into a frequency-dependent direction (see Fig. 2). In order to filter out the stray light scattered off the
cryostat windows, we make the diffracted beams converge to a pin hole with the help of the optical relay formed by the lenses L2 and L3. The pin hole is sized to the dimensions of the active area image formed through lenses L2 and L3. Emerging from this precisely defined emitting area, the diffracted beams are collected on a photodiode array (PDA) through the lens L4 operating as an angle-to-position Fourier transformer. The PDA includes 1024 pixels. The pixels are 2.5 mm high and 25 μm wide. They are sequentially read out every 10 ms. Since the detector is continuously operating, this interval also represents the integration time.

The illuminated position coordinate on the PDA is proportional to the diffraction angle and so to the RF signal frequency. Specifically, a beam diffracted in direction φ hits the PDA at position

\[ x - x_0 = (φ - φ_0) \times \frac{f_4}{f_2} / f_3, \]

where the focal length of lens L4 is denoted by \( f_4 \). The direction φ is connected to the RF frequency by

\[ φ - φ_0 = (ν - ν_0) \times \frac{φ}{\bar{ν}} \]

where \( \bar{φ} \) and \( \bar{ν} \), respectively, represent the angular scanning range of beam #2 and the frequency chirp range of the laser during the engraving step. The spectral channel number of the device is given by the number of different directions that can be addressed by the AO deflector (AOD). Detailed calculation of the angular resolution and the channel number is performed in the appendix.

3.2. Experimental results

Using this setup we demonstrate a 3.3 GHz instantaneous bandwidth with 100-channel resolution. The data are displayed in Fig. 3c, together with two previous stages of the spectrum analyzer demonstration [13,14]. All the experiments have been performed with the same 2.5 mm-thick Tm\(^{3+}\):YAG (0.5 at%) crystal.

In a previous publication [15], we demonstrated the spectral analysis of an RF signal that was put on the probe beam with the help of a Mach–Zehnder modulator. In the present work, we simulate a multiple-line RF signal by making the laser perform various discrete frequency jumps during the readout step of the writing/readout sequence. The staircase frequency scan of the probe beam is illustrated in Fig. 4. The laser stays for 10 μs at each frequency step.

In order to increase the intensity, we have reduced the vertical spot size to 150 μm. With a beam waist at the deflector \( w_{AO} = 1.5 \) mm, and a magnification factor \( m_x = 0.25 \), the horizontal spot size is 750 μm. The fixed engraving, deflected engraving, and probe beam power, respectively, amount to 4.7, 3.2 and 0.7 mW.

Given the acoustic wave velocity \( v = 650 m/s \), the acoustic wave chirp rate \( r_A = 1.2 \times 10^{11} s^{-1} \), the calculation in the appendix predicts an angular resolution \( \delta φ = 1.64 \) mrad (see Eq. (A.14)), which corresponds to \( \sim 5 \) pixels of the PDA. We actually measure an angular resolution of \( \sim 6 \) pixels.

Combined with a 205 mrad scanning range, this resolution offers a 100-channel capacity. The angular scanning range is accidentally limited to 205 mrad because of a beam folding mirror size. The XY AOD actually offers more than 270 mrad scanning range, which corresponds to 135 channels. Compensation of the acousto-optic lens effect (see the appendix) would probably increase this number by reducing the channel width.

4. Proceeding to 100% duty cycle operation

In order to continuously detect the optically carried RF signal, without dead time, one has to
refresh the engraved gratings and to analyze the
signal simultaneously. The single laser we use in
the present setup is not able to accomplish these
two tasks. Two different lasers are needed since the
engraving beam frequency has to be continuously
scanned at fixed rate, while the RF signal to be
investigated must be transferred onto a fixed
frequency carrier. The relative stability of the
two lasers should be better than the expected
resolution of the spectrum analyzer. In order to
proceed toward 100% duty cycle operation, we
have investigated an active stabilization scheme
where the laser frequencies are periodically com-
pared after each frequency scan of the engraving
laser.

We have built two identical extended cavity
diode lasers with intracavity electro-optic crystal.
As sketched in Fig. 5, one laser is frequency
scanned every 500 µs. The other laser is left free
running. A fast pre-amplified photodiode (Electro-
Optics Technology Inc., ET2030-A, 1.2 GHz
bandwidth, +26 dB preamp) detects the laser beat
during a short (∼50 µs) standby time interval we
set between two frequency scans. The beat signal,
ranging up to 1 GHz, is first mixed with a local
oscillator (Marconi Instruments Signal Generator
2022D) that can be tuned from 0 to 1 GHz. This
provides us with wide tunability. Then the down-
shifted signal frequency is measured directly by a
12-bit binary counting unit. Counting is performed
during the 50 µs sampling interval where both
lasers are free running. At ∼70 MHz central
frequency, the 50 µs sampling interval accommod-
ates 3500 oscillations. The counter offers an
upper frequency limit of 100 MHz. The 12-bit

Fig. 4. Three successive stages of the RF spectral analyzer
demonstration. (a) narrow band demonstration [13], (b) first
broadband setup [14] and (c) broadband setup equipped with a
5-mm-aperture deflector.

Fig. 5. Time diagram of the engraving and probe laser
evolution. The engraving sequence is repeated at
2 kHz rate. The engraving and probe frequencies are compared
during a 50 µs interval every 500 µs.
counter is an assembly of three SN74F161 4-bit binary counters. The 8 higher weight bits are fed to a AD7524 fast digital-to-analog converter. Then the analog signal is integrated with a time constant that should be larger than the 500 μs separation of two sampling steps, and finally fed to the intracavity EO crystal of the free-running laser.

Operating at a central frequency of ~70 MHz, the device offers a ~60 MHz capture range and bandwidth. To test the relative frequency stability of the locked lasers, we have recorded the two laser beat with the help of a Tektronix TDS3032 oscilloscope. The scanned laser frequency undergoes a 450 μs-long linear chirp over a 1.25 GHz interval, 2000 times/s. The two laser beat is recorded for 10 μs out of the 50 μs time interval where the engraving laser frequency is steady before each scan. To be recorded, the laser beat frequency is downshifted to ~12 MHz. Therefore, the 500-sample recording is comprised of ~100 periods. Recording is repeated at random for several minutes. The averaged signal spectrum is displayed in Fig. 6 in logarithmic scale. The FWHM turns out to be significantly smaller than 1 MHz.

With a time constant of ~10 ms, the locking device only controls the long-term drift of the lasers. The beat frequency measurement is effected at 2 kHz repetition rate and gives some information on the laser passive stability on shorter time scale. This appears to be better than 1 MHz. The data also indicate that the scanned laser returns to its initial frequency with excellent precision. This will allow us to make the two locked lasers respectively play the role of engraving lasers and probe sources. However, as it will become clear in the next section, 2D operation of the spectrum analyzer is more exacting in terms of laser stability. In the 2D operation context, the chirp itself needs to be stabilized. This will demand a more elaborate locking procedure.

5. Scenario for 1000-channel demonstration

5.1. Two-dimension operation of the spectrum analyzer

The number of resolved spectral channels is closely related to the time × bandwidth product of the AOD. Indeed this product represents the number of different directions that can be addressed by the AOD. Used with a Gaussian beam expanded to the maximum held diameter of 4.2 mm at 1/e², and operated over a 40 MHz acoustic wave bandwidth, the A&A DTS XY 250 deflector offers a time × bandwidth product of ~350. A larger device with a clear aperture of ~14 mm would be needed to reach a 1000-channel resolution. This is a straightforward way to achieve the assigned goal. However, one should remember that, in order to compensate for the acoustic wave frequency shift incurred by the deflected light field, we are using a two-deflector XY device, the light beam being successively diffracted in orders +1 and −1 by the two AO crystals. The above-mentioned 350 time × bandwidth product corresponds to scanning along a diagonal of the accessible solid angle. The device is able to address more than 60,000 different directions in 2D scanning. We should take advantage of the large amount of available 2D angular channels, keeping in mind that an attractive feature of the SHB spectral analyzer is its 2D operation ability.

The 2D operation is illustrated in Fig. 7. The detection solid angle and the angular region spanned by the deflected engraving beam are interlaced, which enables us to reduce the angular
distance of $\mathbf{k}_2$ and $-\mathbf{k}_3$ and to somehow relax the phase matching condition (arrangement suggested by Wagner [20]). In Section 5.2, we express the number of available channels as a function of the AOD parameters and we consider the geometric limitations imposed by the beam overlap in the sample and by the phase matching condition. In Section 5.3, we address the engraving beam detuning problem that arises from imperfect acoustic frequency shift compensation when the deflector operates off the diagonal region. In Section 5.4 we report on the first experimental investigation in the prospect of 2D operation.

5.2. Geometric limitations

The angular arrangement of the engraving, probe and diffracted beams is sketched in Fig. 8. The deflected engraving beam direction $\mathbf{k}_1$ is defined by the angular coordinates $(\theta, \phi)$ with respect to the fixed engraving beam that propagates along $\mathbf{k}_2$. The deflected engraving beam scanning spans the angular intervals $\bar{\phi}$ and $\bar{\theta}$ along horizontal and vertical directions, respectively. At some appropriate position, most likely at the Fourier plane position with respect to the active crystal through a focusing lens, a multiple slit reflecting mask is used to separate the scanned engraving beam from the diffracted signal. One directs the engraving beam through the horizontal slits and has the diffracted signal been reflected on the spacing between the slits. The reverse situation could be preferred. Let $\delta \phi$ and $\zeta$, respectively, represent the horizontal and vertical angular size of a spectral channel. Then the channel number along a horizontal line is given by $N_x = \bar{\phi}/(2\phi)$. Every second line is occupied by the deflected engraving beam. Therefore, the number of lines that accommodate the signal is $N_y = \bar{\theta}/(2\zeta) + 1$. The probe beam wave vector $\mathbf{k}_3$ is set at angular distance $\zeta$ from $\mathbf{k}_2$, just below it.

To define the angular size of a channel, we consider Gaussian engraving beams. Their $1/e^2$ waist radius at the active medium position is given by $w_x$ and $w_y$ along horizontal and vertical directions, respectively. The beam crossing the XY AOD is supposed to be axially symmetric so that $w_y$ and $w_x$ are connected to $w_{AO}$, the waist radius at the AOD through the equations

$$
\begin{align*}
w_x &= m_x w_{AO}, \\
w_y &= m_y w_{AO},
\end{align*}
$$

where $m_x$ and $m_y$ stand for the imaging optics magnification factors along vertical and horizontal directions. We define $\zeta$ as the $1/e^2$ angular diameter of a diffracted Gaussian probe beam. According to Eq. (A.14) in the appendix, $\zeta$ is given by

$$
\zeta = \sqrt{\frac{2}{\ln 2}} \frac{2 \sqrt{3}}{\pi} \frac{\lambda}{w_y}.
$$

\[\text{Fig. 7. Two-dimensional operation of the spectrum analyzer. One reduces the angular distance of } \mathbf{k}_2 \text{ and } -\mathbf{k}_3 \text{ by interlacing the engraving and diffracted beams. As a consequence, the phase matching condition is less severe.}
\]

\[\text{Fig. 8. Angular arrangement of the beams. The hatched rectangles represent the directions spanned by the deflected engraving beam. For each position of the deflected beam, the four beam wave vectors define a parallelogram that turns into a rectangle when } \mathbf{k}_2 - \mathbf{k}_1 \text{ is horizontal. This is the only situation when the phase matching condition is exactly satisfied.}
\]
This rather stringent definition guarantees that most of the signal is collected on the above-mentioned reflecting mask. It should be noticed that, at the mask position, the angular size of the engraving beam is $\sqrt{3}$ times smaller than that of the diffracted beam. As a result, the engraving beam will not be stopped by the diffracted beam collecting mask.

We suppose that the channel width along an horizontal line is given by the FWHF. This is very close to the Rayleigh criterion for the resolution of an optical system, the main difference being that Rayleigh considers Airy patterns of diffraction, while we are considering Gaussian spots. Therefore, $\delta\phi$ reads as

$$\delta\phi = \frac{\sqrt{6}\ln 2}{\pi} \frac{\lambda}{w_x}. \tag{3}$$

Let $\tilde{\theta}_{AO} = m_x\tilde{\theta}$ and $\tilde{\varphi}_{AO} = m_y\tilde{\varphi}$ represent the angular spans at the AOD leading to the angular spans $\tilde{\theta}$ and $\tilde{\varphi}$ at the crystal. Making use of $N_x$, $N_y$, $\zeta$ and $\delta\phi$ definitions one obtains

$$N_x = \frac{\pi}{\sqrt{6}\ln 2} \frac{w_{AO}}{\lambda} \tilde{\theta}_{AO},$$

$$N_y = \frac{\pi}{4\sqrt{3}} \frac{w_{AO}}{\lambda} \tilde{\varphi}_{AO} + 1. \tag{4}$$

As expected, the number of channels does not depend on the magnification factor of the optics inserted between the AOD and the active crystal.

However one is not free to elect any $m_x$ and $m_y$ values. Two conditions, namely the phase matching condition and the beam overlapping condition impose $w_x$ and $w_y$ values, once one has fixed $N_x$ and $N_y$ by setting $w_{AO}$, $\tilde{\theta}_{AO}$ and $\tilde{\varphi}_{AO}$ values.

The phase matching condition reads as

$$|(\tilde{k}_2 + \tilde{k}_3) \cdot (\tilde{k}_2 - \tilde{k}_1)| < \pi k / L, \tag{5}$$

where $L$ represents the sample thickness. The condition is exactly satisfied when $\tilde{k}_2 + \tilde{k}_3$ is orthogonal to $\tilde{k}_2 - \tilde{k}_1$. Otherwise it reads as

$$\zeta \sqrt{\varphi^2 + \theta^2} \cos \psi < \lambda / (2L), \tag{6}$$

where $\cos \psi = \theta / \sqrt{\varphi^2 + \theta^2}$. The maximum size of $\theta$ is $\tilde{\theta}/2$. Therefore, the condition reads as

$$\tilde{\theta}_c < \lambda / L. \tag{7}$$

The phase matching condition is finally expressed as

$$w_y > 4 \text{DOF} \sqrt{\frac{3}{\pi}} (N_y - 1), \tag{8}$$

where $w_{\text{DOF}} = \sqrt{\lambda L / 2\pi}$ is the waist dimension when the crystal thickness equals the depth of field (i.e. twice the Rayleigh range) of the focusing optics.

The engraving beam maximal horizontal angular separation, denoted as $\varphi_{\text{max}}$, should be small enough so that the beams overlap all along the light path within the crystal. This condition reads as

$$\varphi_{\text{max}} < 2 \sqrt{\frac{w_x}{L}}, \tag{9}$$

With the help of Eq. (9) and of the definition of $N_x$ one finally expresses the engraving beam overlap condition as

$$w_x > (3\ln 2)^{1/4} w_{\text{DOF}} \sqrt{\frac{\varphi_{\text{max}}}{\varphi}} N_x, \tag{10}$$

where $\varphi_{\text{max}} / \varphi \approx 1$.

Combining Eqs. (8) and (10), one concludes that, for $N$-channel operation, the active area should spread over a surface at least equal to $\sim 5 (w_{\text{DOF}})^2 \sqrt{N}$. Owing to the non-linear character of the four-wave mixing process, one has to focus the engraving beams as close as possible to the minimum size in order to optimize the energy budget.

### 5.3. Signal formation dynamics

In this section, we examine the requirements for 2D operation in terms of dipole lifetime and laser stability. We determine the law of variation of the signal intensity as a function of the detuning of the two AOs. We deduce the $N_y$ dependence of the signal intensity.

Let $\theta_{AO}$ and $\varphi_{AO}$ denote the vertical and horizontal deflection at the AOD plane. As a function of the $X$ and $Y$ AOD driving frequencies $f_1$ and $f_2$, the deflection angles read as

$$\theta_{AO} = \frac{\lambda f_1 - f_2}{v \sqrt{2}},$$

$$\varphi_{AO} = \frac{\lambda f_1 + f_2}{v \sqrt{2}} \tag{11}$$
The total shift vanishes along the horizontal diagonal $\theta_{AO} = 0$ where $f_1 = f_2$. If $f_1 \neq f_2$, each atom successively interacts with the two engraving beams. The time interval of those interactions is given by $t_{12} = (f_1 - f_2)/r$, where $r$ denotes the laser chirp rate.

When the spectrum analyzer is operated along a single line (1D operation), the detuning is adjusted in such a way that $t_{12}$ is larger than the inverse spectral resolution (see the appendix). In a 2D spectrum analyzer, one has to increase the detuning beyond the inverse spectral resolution in order to explore the off-diagonal region of the accessible solid angle. During the time interval $t_{12}$, the memory of interaction is carried by an atomic superposition state. The phase that builds up during this time interval fluctuates with the laser frequency the more as $t_{12}$ is larger [19], which tends to ruin accumulated engraving. Therefore, with increasing line number $N_y$ the device is more and more demanding in terms of optical dipole lifetime and laser stability. It should be stressed that in this instance the dipole and laser stability are not spent to improve the resolution of the spectrum analyzer but to increase the line number. A 1D device offering 100 channels over 1 GHz exhibits the same resolution as a 2D device offering 1000 channels over 10 GHz, with 10 lines of 100 channels each. However, the latter requires a laser 10 times more stable and a dipole lifetime 10 times larger than the former.

According to Eqs. (2) and (11), the increment of $f_1 - f_2$ between two lines is

$$\delta f = \frac{4\sqrt{6}}{\pi} \frac{v}{w_{AO}}.$$  \hspace{1cm} (12)

According to Eq. (A.17), the diffraction intensity in 1D operation reads as

$$S(v, \varphi, 0) = [1 + \text{erf}(u)]^2 e^{-4t_{12}/T_2},$$  \hspace{1cm} (13)

where

$$u = \frac{t_{12}}{[t_{12}]_{\text{min}}} - \frac{[t_{12}]_{\text{min}}}{T_2}$$

and

$$[t_{12}]_{\text{min}} = \frac{\varphi \sqrt{2} \ln 2}{\pi v \delta \varphi}.$$  \hspace{1cm} (14)

Let $n_y$ represent the line label ranging from 1 to $N_y$ in 2D operation. We assume that the delay $t_{12}$ is adjusted to $[t_{12}]_{\text{min}}$. Then the delay along line $n_y$ is expressed as

$$t_{12} = [t_{12}]_{\text{min}} + (n_y - 1) \frac{\delta f}{r}.$$  \hspace{1cm} (15)

The diffraction intensity $S(v, \varphi, 2(n_y - 1)\zeta)$ along line $n_y$ is given by Eq. (13) where Eq. (15) has been substituted for $t_{12}$. The variation of $S(v, \varphi, 2(n_y - 1)\zeta)$ as a function of $n_y$ and $t_{12}$ is displayed in Fig. 9. We assume that $N_x = 200$, which is consistent with our XY AOD aperture. Then, to get 1000-channel resolution we set $N_y = 5$. The engraved gratings are supposed to be refreshed at a repetition rate of 4 kHz and the total spectral bandwidth is set equal to 10 GHz. Therefore, the scanning duration of each horizontal line equals 250 $\mu$s/5 = 50 $\mu$s and the scanning range of each line reads $\varphi = \frac{\pi}{2} = 2$ GHz. The chirp rate is $r = 2$ GHz/50 $\mu$s = $4 \times 10^{13}$ s$^{-1}$. With $w_{AO} = 2$ mm, $v = 650$ m/s and $T_2 = 1$ $\mu$s, the signal intensity varies by less than 20% over the entire frequency range. We have ignored the laser frequency fluctuations. To check the validity of

![Fig. 9. Two-dimensional operation of the spectrum analyzer.](image)
the analysis we experimentally investigate the diffracted signal variations as a function of \( f_1 - f_2 \).

5.4. Experimental investigation

In a first set of experiments, we measure the variations of a single line signal intensity as a function of \( f_1 - f_2 \) for various values of \( r \). Results are displayed in Fig. 10. To compare experimental results with theoretical predictions, we fit the profile described by Eq. (13) to the data by adjusting the optical dipole lifetime value, all other parameters being fixed. The best fit value of \( T \) appears to depend on the laser chirp rate, varying from 0.25 \( \mu \)s at \( r = 2 \times 10^{12} \text{ Hz/s} \) to 0.11 \( \mu \)s at \( r = 8 \times 10^{12} \text{ Hz/s} \). These values are much smaller than the expected optical dipole lifetime. Above all, the \( r \) dependence of the obtained \( T \) values is enough to rule out the atomic origin of the decay. Laser frequency fluctuations may cause this decay. Indeed, they would make the phase of the engraving beam interference fringes fluctuate from shot to shot, thus preventing the build up of the accumulated pattern.

![Graph](image_url)

**Fig. 10.** Diffracted signal intensity variations as a function of the acousto-optic driver detuning \( f_1 - f_2 \). The experimental data correspond to \( \gamma = 0.5 \text{ GHz}, r = 2 \times 10^{12} \text{ Hz/s} \) (rectangles); \( \gamma = 1 \text{ GHz}, r = 4 \times 10^{12} \text{ Hz/s} \) (diamonds); and \( \gamma = 2 \text{ GHz}, r = 8 \times 10^{12} \text{ Hz/s} \) (triangles). Results are displayed as a function of \( t_{12} = (f_1 - f_2)/r \). Fitting Eq. (13) to the data leads to \( T_1 = 0.25 \mu \text{s at } r = 2 \times 10^{12} \text{ Hz/s}, T_1 = 0.20 \mu \text{s at } r = 4 \times 10^{12} \text{ Hz/s}, \) and \( T_2 = 0.11 \mu \text{s at } r = 8 \times 10^{12} \text{ Hz/s} \). The fitted curves are drawn as solid lines.

To clarify the origin of the decay, we have compared the single shot process with the accumulated regime. In this experiment, the XY AOD drivers operate at fixed frequency, with provision for an adjustable detuning between them. In other words, the engraving beam angle of incidence is not scanned in synchrony with the laser chirp. Therefore, the diffracted beam is detected in fixed direction and we have replaced the photodiode array by a single avalanche photo detector that provides us with time domain discrimination of the signal against spurious light. This arrangement proves useful since the single shot signal is about 40 times smaller than the signal diffracted on an accumulated grating. Results are displayed in Fig. 11. They confirm that decay in the single shot operation is consistent with the expected optical dipole lifetime. We ascribe the fast signal decay as a function of \( t_{12} \), as observed in the accumulation regime, to the shot-to-shot fluctuation of the spectral grating phase.

To relate this effect to laser fluctuations, we rely on the phase diffusion model. The chirped laser field is described as

\[
E(t) = e^{2i\nu_0 t + i\nu t^2 + i\delta\varphi(t)},
\]

where \( r, f \) and \( \delta\varphi(t) \), respectively, stand for the frequency chirp rate, the acousto-optic controlled detuning of the two fields and the phase fluctuation. All phase fluctuations are collected in \( \delta\varphi(t) \) so that \( r \) is considered as constant. Within the frame of the phase diffusion model, phase fluctuation simply leads to the substitution of the decay rate \( 1/T_2 \) with

\[
\frac{1}{T_2} = \frac{1}{T_2} + \frac{1}{2\gamma},
\]

where \( \gamma \) represents the half-width at half-maximum of the spectrum of a fixed-frequency laser affected by the same phase fluctuation pattern. Analyzing Fig. 10 within the frame of this model, we conclude that \( \gamma \) varies as a function of the chirp rate, strongly increasing when \( r \) is varied from \( 2 \times 10^{12} \) to \( 8 \times 10^{12} \text{ Hz/s} \). From the data displayed in Fig. 11, we obtain \( \gamma = 1.35 \text{ MHz} \) at \( r = 4 \times 10^{12} \text{ Hz/s} \). This value leads to a laser line FWHM of \( \sim 2.7 \text{ MHz} \), about three times larger.
than the two-laser beat spectrum width given in Section 4.

The chirp dramatically deteriorates the laser stability. We have to fix this problem before implementing the spectrum analyzer 2D operation. Instead, we might compensate for the undesired frequency detuning of the engraving beams with the help of an additional pair of acousto-optic modulators, working as a frequency shifter. In that way, we could reduce the delay $t_{12}$ to the size of the inverse spectral resolution, as in 1D operation. However, this is a complex, ponderous and energy wasting solution. In the more fundamental prospect of laser noise investigation, the influence of chirp rate on the laser stability has to be examined more carefully. The experimental data clearly show that the laser line width increases with the chirp rate but this connection is not accounted for by the phase diffusion model.

6. Conclusion

An SHB spectrum analyzer with 3.3 GHz bandwidth and 100 channels (33 MHz each) has been experimentally demonstrated. The goal is to demonstrate 10 GHz bandwidth with 1000 channels and 100% duty cycle. We have shown that a fixed-frequency laser, meant for the role of RF signal carrier, can be referenced efficiently to the continuously swept writing laser. This is a step towards 100% duty cycle. We have introduced the new 2D operation concept that opens the way to increased channel capacity. Experimental work will aim at proving this in the future. The XY AO deflector offers a direct way to achieve the 2D addressing of the spectral channels. However, the resulting detuning of the engraving fields makes the accumulated gratings much more sensitive to the laser stability, especially during the frequency chirp. This illustrates the importance of the laser quality in such applications and the general need for developing high-stability laser technology.

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Appendix. Signal calculation

We calculate the diffracted signal intensity. This provides us with the expression of the spectral/angular resolution as a function of the laser beam waist size. We also obtain the diffracted signal intensity variation as a function of the frequency detuning of the two engraving beams. Calculation is performed within the limits of the following assumptions:

(i) Gaussian beams: The three beams are given the same waist $w$.

(ii) Optically thin sample: Engraving is uniformly laid within the photosensitive plate thickness and neither the probe field nor the signal are attenuated while propagating through it.
(iii) Engraving accumulation by optical pumping to the shelving state is supposed not to modify either the spatial or the spectral distribution of the engraving with respect to a single-shot operation.

(iv) Lowest-order perturbation limit: The absorption coefficient modification is proportional to the power spectrum of the engraving fields. The same approximation applies to the readout field.

Let \( \vec{k}_1 \) denote beam \# 1 wave vector at time \( t = 0 \). During engraving by fields \( E_1 \) and \( E_2 \) the wave vector \( \vec{k}_2 \) is kept fixed while the wave vector of beam \# 1 is scanned around the \( O_y \) axis, perpendicular to the \( (\vec{k}_1, \vec{k}_2) \) plane, by the AO deflector assembly represented in the inset of Fig. 2. The phase shift imparted at position \( \vec{r} \) in by the acoustic wave propagating in deflector \( D_j \) obeys the following relation:

\[
\Phi_j(\vec{r}, t) = \Phi_j \left( 0, t - \frac{\vec{k}_j \cdot \vec{p}}{v} \right),
\]

where \( \vec{k}_j \) and \( v \), respectively, represent the sound wave unit vector and the sound velocity. The input phase shift reads as

\[
\Phi_j(0, t) = 2\pi \int_0^t f_j(t') \, dt',
\]

where the linearly chirped acoustic frequency \( f_j(t) \) is expressed as

\[
f_j(t) = f_j^0 + r_A t,
\]

where \( f_j^0 \) and \( r_A \) stand for the AOM central frequency and the chirp rate, respectively. The sound wave unit vectors are oriented at 45° from the deflection axis \( O_x \), directed along \( \vec{R} = \vec{k}_2 - \vec{k}_1 \), and they are orthogonal to each other. The combined phase shift imparted to \( E_1 \) is

\[
\Phi(\vec{r}, t) = \Phi_1(\vec{r}, t) - \Phi_2(\vec{r}, t)
\]

\[
= 2\pi \left[ -\frac{x}{v\sqrt{2}} (f_1^0 + f_2^0) - \frac{y}{v\sqrt{2}} (f_1^0 - f_2^0) + (f_1^0 - f_2^0)t - \sqrt{2} \frac{r_A}{v} xt + \frac{r_A}{v^2} xy \right].
\]

The first two terms on the right-hand side, respectively, represent the horizontal and vertical deflection at time \( t = 0 \). The third term reflects the optical frequency shift conveyed by the deflector pair. Therefore the \( v_1 - v_2 \) relative frequency shift of the engraving beams is constant and given by

\[
v_1 - v_2 = f_1^0 - f_2^0.
\]

The fourth term in Eq. (A.4) corresponds to the time-dependent angular deflection. The last term is a time-independent, purely spatial quadratic phase. In other words, a chirped acousto-optic deflector behaves as a lens, the focal length of which depends on the chirp rate. An AO deflector would mimic a cylindrical lens since it operates along a single direction. In our setup, this quadratic phase factor modifies imaging conditions and damages the spectral resolution.

The first two terms in the phase expression (Eq. (A.4)) are incorporated in the wave vector \( \vec{k}_1 \) at \( t = 0 \). The laser frequency is swept at rate \( r \) in synchrony with the angular scan. Then the relevant Rabi frequencies read as

\[
\chi_1(\vec{r}, t) = \chi_0(\vec{p}) e^{i \pi r^2 - 2i \pi (r_A/v)x t + 2i \pi (r_A/v^2)xy},
\]

\[
\chi_2(\vec{r}, t) = \chi_0(\vec{p}) e^{i \pi r^2 - 2i \pi (v_1 - v_2) r - i \vec{k}_2 \cdot \vec{r}},
\]

where the Gaussian beam Rabi frequency spatial distribution is expressed in terms of the beam waist at the AOD, \( w_{A0} \), as

\[
\chi_0(\vec{p}) = \chi_0(0) \exp \left( -\frac{\vec{p}^2}{w_{A0}^2} \right).
\]

Low-intensity engraving can be described in terms of the incident light power spectrum. The spectral amplitude of the field Rabi frequency is

\[
\tilde{\chi}_1(\vec{p}, \delta) = \sqrt{\frac{i}{\pi}} \chi_0(\vec{p}) \times e^{-\frac{\pi \delta^2}{r^2 - 2i \pi \delta (v_1 - v_2)}} \left( r_A \sqrt{2/r} + 2i \pi (r_A/v^2)xy - 2i \pi (r_A/v^2)yt \right),
\]

\[
\tilde{\chi}_2(\vec{p}, \delta) = \sqrt{\frac{i}{\pi}} \chi_0(\vec{p}) e^{-\frac{\pi \delta^2}{r^2 - r_A^2 - i \vec{k}_2 \cdot \vec{r}}},
\]

The AOD is imaged on the active crystal with the magnification factors \( m_x \) and \( m_y \) along directions \( x \) and \( y \), respectively. Under illumination by the engraving fields, the ion population distribution
which combines the causality-allowed contribution first leads to the expression at the crystal. Performing the space coordinates \( X \) and \( Y \) where the laser spot undergoes a modification proportional to the exciting field power spectrum \( |\tilde{Z}_1(\tilde{p}, v - v_1) + \tilde{Z}_2(\tilde{p}, v - v_1)|^2 \), convolved by the homogeneous line profile \( L(v) \). The contribution of the cross-term \( \tilde{Z}_1^*(\tilde{p}, v - v_1)\tilde{Z}_2(\tilde{p}, v - v_1) + \text{c.c.} \) gives rise to the diffraction gratings dedicated to the spectral analysis of the optically carried RF signal.

Let a monochromatic Gaussian probe beam at frequency \( v \) be directed to the sample along \( \vec{K}_3 \). The probe beam is assumed to exhibit the same spatial frequency component when space coordinates \( X \) and \( Y \) are now expressed at the crystal. Performing the space integration first leads to

\[
S(v, \varphi, \theta) = \left| \tilde{E}(v, \varphi, \theta) \right|^2 = \\
\left| \int \int dY \int dX \tilde{E}(\tilde{p}, v) e^{i(\vec{k}_3 + \vec{K}) \cdot \tilde{p}} e^{(2in/\lambda)(\phi X + \theta Y)} \right|^2,
\]

(A.10)

where space coordinates \( X \) and \( Y \) are now expressed at the crystal. Performing the space integration first leads to

\[
S(v, \varphi, \theta) = \left| \tilde{E}(v, \varphi, \theta) \right|^2 = \\
\left| \int dY \int dX \tilde{E}(\tilde{p}, v) e^{i(\vec{k}_3 + \vec{K}) \cdot \tilde{p}} e^{(2in/\lambda)(\phi X + \theta Y)} \right|^2,
\]

(A.11)

which combines the causality-allowed contributions from all the atoms. In the frame of the Huygens–Fresnel principle Fraunhofer approximation, the signal emitted at angular distance \((\varphi, \theta)\) from direction \( \vec{K}_3 + \vec{K} \) is readily obtained as the following Fourier transform of \( E(\tilde{p}, v) e^{i(\vec{k}_3 + \vec{K}) \cdot \tilde{p}} \):

\[
E(\tilde{p}, v, \varphi, \theta) = \int dY \int dX \tilde{E}(\tilde{p}, v) e^{i(\vec{k}_3 + \vec{K}) \cdot \tilde{p}} e^{(2in/\lambda)(\phi X + \theta Y)},
\]

(A.12)

where

\[
g(v, \varphi, \theta) = e^{-2Ln2\times[(\tilde{v}\varphi/\tilde{v}+\varphi)|\delta\varphi|^2+(\theta/\delta\theta)|^2]},
\]

(A.13)

\[
\delta\varphi = \frac{\sqrt{6Ln2}}{\pi} \frac{\lambda}{m_v w_{AO}} \sqrt{1 + z^2},
\]

(A.14a)

\[
\delta\theta = \frac{\sqrt{6Ln2}}{\pi} \frac{\lambda}{m_p w_{AO}} \sqrt{1 + z^2},
\]

(A.14b)

\[
z = \pi R_{AO}^2 / \nu^2,
\]

(A.15)

\( \tilde{v} \) represents the frequency interval over which the laser is scanned while \( \varphi \) is varied over the angular range \( \tilde{\varphi} \) and \( T_2 \) stands for the optical dipole lifetime. If \( v_{12} \gg \tilde{\varphi}/\delta \varphi \) the integrant in Eq. (A.12) vanishes in the \( \tau < 0 \) region. Therefore, the lower boundary of the integral over \( \tau \) can be shifted to \(-\infty\) and the signal intensity reduces to

\[
S(v, \varphi, \theta) = e^{-4v_{12}/T_2 [g(v - v_1, \varphi, \theta)]^2}.
\]

(A.16)

This is the well-known photon echo situation that leads to optimal spectral resolution by eliminating the dispersive part of the diffracted signal [21]. According to Eq. (A.16), the spectral component \( v_1 \) is diffracted in direction \((\varphi = 0, \theta = 0)\). The angular FWHM of the diffracted beam is given by \((\delta\varphi, \delta\theta)\). The spectral component \( v \) is diffracted in direction \((\varphi = (v - \tilde{v})/\tilde{v}, \theta = 0)\), with the same angular spreading as component \( v_1 \). The angular spreading of each frequency channel limits the spectral resolution of the spectrum analyzer to \( \delta v = \tilde{v} \delta\varphi/\tilde{\varphi} = \tilde{v}(v/w_{AO})/(\sqrt{6Ln2}/\pi)\sqrt{1 + z^2} \), where \( \tilde{f} \) stands for the acoustic frequency scanning range of the AOD. The frequency resolution is independent of the optics magnification factor. It only depends on the engraving laser scanning range and on the number of directions that can be addressed by the AOD.

The parameter \( z \) characterizes the AOD lens effect. On account of the AOD driver frequency chirp, the two sides of the laser spot undergo deflection at different angles in the AOD. The angle difference can be expressed in terms of the chirp rate and of the acoustic wave transit time across the light beam, \( w_{AO}/v \), as \( \lambda_{AO}/w_{AO}/v^2 \). This angle has to be compared with the divergence of the laser beam, \( \lambda/w_{AO} \). The parameter \( z \) is nothing but the ratio of these two quantities. The lensing effect is negligible when \( \lambda_{AO}/w_{AO}/v^2 \ll \lambda/w_{AO} \). The channel broadening caused by the acousto-optic deflector pair lens effect is not a fundamental limitation. On account of the holographic storage properties, the corresponding wave front distortion does not entail irreversible signal alteration. The quadratic cross phase factor can be alleviated

\[
\delta\varphi = \frac{\sqrt{6Ln2}}{\pi} \frac{\lambda}{m_v w_{AO}} \sqrt{1 + z^2},
\]

(A.14a)

\[
\delta\theta = \frac{\sqrt{6Ln2}}{\pi} \frac{\lambda}{m_p w_{AO}} \sqrt{1 + z^2},
\]

(A.14b)
by an appropriate non-axially symmetric wave
front correction to any of the four fields involved
in the signal formation.

The last information we derive from Eq. (A.12)
is the signal intensity variation as a function of the
two engraving beam detuning. Integrating
Eq. (A.12) one obtains

\[
S(v, \varphi_0, 0) = [1 + \text{erf}(u)]^2 e^{-4t_2^2/T_2}, \quad (A.17)
\]

where

\[
u = \frac{\pi \varphi \alpha \delta \varphi}{\varphi \sqrt{2 \ln\varphi}} = \frac{\sqrt{2 \ln\varphi}}{\pi \varphi}
\]

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