Theoretical and experimental analysis of intensity noise correlations in an optically pumped, dual-frequency Nd:YAG laser

Syamsundar De,1 Goulc’hen Loas,2 Abdelkrim El Amili,2 Mehdi Alouini,2 and Fabien Bretenaker1,*

1Laboratoire Aimé Cotton, CNRS-Université Paris Sud 11-ENS Cachan, Campus d’Orsay, 91405 Orsay, France
2Institut de Physique de Rennes, CNRS-Université de Rennes 1, Campus de Beaulieu, 35042 Rennes, France
*Corresponding author: Fabien.Bretenaker@u-psud.fr

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The spectral behavior of the intensity noise correlations between the two orthogonally polarized modes of a dual-frequency Nd:YAG laser is investigated, both experimentally and theoretically. We show, for different coupling strengths between the two orthogonally polarized modes, how the presence of the relaxation oscillations due to the class B dynamical behavior of our laser affects the shape of the intensity noise correlation spectra. Different coupling situations have been achieved by controlled alignment of the two orthogonally polarized oscillating modes with respect to the crystallographic axes of a (100)-cut neodymium-doped yttrium aluminum garnet active medium in which the light-emitting dipoles behave as if they are aligned along the crystallographic axes. The theoretical modeling has been done based on the assumptions that the only source of noise for the two lasing modes in the interesting frequency range (0–120 kHz) is the intensity noise of the pump diode laser, the pump noises for the two modes are white frequency noises of identical amplitudes, and the pump fluctuations are in phase, but partially correlated. © 2013 Optical Society of America

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1. INTRODUCTION

Metrology, remote sensing, or communication applications, as well as wideband radar signal processing [1] and long-range transmission of high-purity radio frequency (RF) references [2], require direct generation of tunable, optically carried RF signal. The obvious technique to generate such optically carried RF signals is optical mixing of two independent lasers [3] but, unfortunately, the large linewidth of the beat-note limits this technique for the above-mentioned applications. One interesting way to generate an optically carried RF signal could be a dual-frequency laser in which two orthogonally polarized modes oscillate inside the same cavity and the frequency difference between the two modes is in the RF range and tunable. The advantages of this technique are the narrow linewidth of the RF beat-note due to correlated fluctuations of the two laser modes oscillating inside the same cavity, as well as the large tunability of the beat frequency obtained by varying the intracavity phase anisotropy [4,5]. Such dual-frequency lasers of different wavelengths have been demonstrated for various kinds of solid-state active media having different ion–host combinations, e.g., Nd:YAG [6], Er:Yb:glass [7], Yb3+-KGW [4]. But the main limitations for microwave photonics applications comes from the fact that these solid-state lasers suffer from resonant intensity noise at low frequencies (few kilohertz to few megahertz) due to their inherent class B dynamical behavior [1,8]. For dual-frequency, solid-state lasers sustaining two orthogonally polarized modes, each eigenmode exhibits two peaks in its intensity noise spectrum. The first one is the well-known in-phase (standard) relaxation oscillation frequency, for which the intensities of the two modes relax in phase. The second one, which is inherent to all coupled oscillator systems, corresponds to the antiphase relaxation regime of the intensities of the two eigenstates [9]. The presence of these noise peaks at these two relaxation oscillation frequencies causes severe degradation of the beat-note spectral purity [1,10].

One way to avoid this problem is to develop a dual-frequency class A laser, which will be inherently free from relaxation oscillations. This has been recently demonstrated for dual-frequency vertical external cavity surface emitting lasers (VECSELs) [11]. However, in the case of such a semiconductor dual-frequency laser, the phase noises of the oscillating modes are linked to the intensity noises of these modes via the Henry factor describing the phase/intensity coupling. This is why it is important to understand and control the correlations between the intensity noises of the two modes in such a dual-frequency VECSEL, as recently achieved [12].

Although the phase/intensity coupling is much lower in solid-state lasers than in semiconductor lasers, because the Henry factor is much weaker in the former case, a complete understanding of the noise characteristics of such a dual-frequency, solid-state laser is highly desirable. In all cases, as far as the addressed application requires the use of the two laser modes (atomic clock, pump-probe experiments, metrology, etc.), the degree of intensity noise correlation between the two modes must be known precisely. This is why we wish here to explore the correlations between the intensity noises of the two modes of a dual-frequency, solid-state laser.
More specifically, we focus here on a dual-frequency Nd:YAG laser based on a (100)-cut Nd:YAG crystal. Indeed, in such a crystal, the emitting dipoles can be approximated as if they were aligned along the crystallographic axes of the gain medium [13]. This has recently proved to be useful in controlling the coupling between the two lasing modes by aligning their polarizations along two orthogonal crystallographic axes of the (100)-cut Nd:YAG crystal [14].

The aim of this paper is consequently to study the intensity noise correlation behavior for a dual-frequency Nd:YAG laser for different coupling situations in the same spirit as demonstrated in a dual-frequency VECSEL [12]. But the interesting distinct feature for the dual-frequency Nd:YAG laser is that the noise spectra for the two oscillating modes are expected to be largely modified compared to dual-frequency VECSEL due to the presence of relaxation oscillations because of class B dynamical behavior of such a laser. Moreover, the Nd:YAG laser and VECSEL, respectively, being a solid-state and a semiconductor laser, there is an obvious fundamental difference between the gain mechanisms. In the present paper, we thus investigate, both theoretically and experimentally, the relative intensity noise (RIN) spectra and also the correlation between the intensity noises of the two lasing modes for different coupling situations. In our laser, which is based on a (100)-cut Nd:YAG crystal gain medium, different coupling strengths between the eigenpolarization modes can be achieved by varying the eigenpolarization directions with respect to the crystallographic axes [13,14]. In Section 2, we start by developing a theoretical model. This model is based on the assumption that the dominant source of noise in the interesting frequency range (0–120 kHz), i.e., where the laser intensity noise is well above shot noise, is the pump laser source. Moreover, one assumes that the pump noises seen by the two modes are white noises of identical amplitude and are partially correlated. In Section 3, we describe the experimental setup aimed at measuring the intensity noise correlation between the two orthogonally polarized modes. The comparison of the theoretical model with the experiments for different coupling situations is reported in Section 4. In this section, the results of the pump noise measurements are also depicted, which justify the white frequency pump noise approximation taken for developing our theoretical model. The physical mechanisms behind our results are discussed in Section 5.

2. THEORETICAL MODEL

In this section, we derive a theoretical model to assess how the intensity noise of the pump laser is transferred to the intensity noises of the two orthogonally polarized modes of our dual-frequency Nd:YAG laser. To model our laser, we start from the rate equations governing the time evolution of the photon numbers $F_1$ and $F_2$ in the two modes as developed in [13]. This model is based on the assumption that the transition dipoles are oriented along the three crystallographic axes $(u_1, u_2, u_3)$ and are associated with population inversions $N_{10}$, $N_{02}$, and $N_3$, respectively [Fig. 1(a)]. In our case the Nd:YAG crystal is (100)-cut. In this configuration, the wave vector $k$ of the two lasing modes is aligned along one of the three crystallographic axes (say $u_1$) and the two orthogonal linearly polarized modes, lying in the same plane as the other two transverse crystallographic axes, are defined by

$$\mathbf{x} = \left( \cos \alpha \sin \beta, \sin \alpha \sin \beta, \cos \beta \right), \quad \mathbf{y} = \left( -\sin \alpha \sin \beta, \cos \alpha \sin \beta, \cos \beta \right).$$

where the coordinates have been written in the basis $(u_1, u_2, u_3)$ of the crystallographic axes. The two orthogonal eigenpolarizations, $\mathbf{x}$ and $\mathbf{y}$, make angle $\alpha$ with crystal axes $u_1$ and $u_2$, respectively [Fig. 1(b)]. The dipoles, aligned along crystallographic axes, possess a small ellipticity $\beta \ll 1$, probably because of the existence of arbitrary orientation of the Nd$^{3+}$ ions, which are residing on the defect sites of the YAG matrix and inducing energy transfer between dipoles from different intrinsic crystal sites. So, as mentioned in [13], the rate equations for our laser, sustaining two orthogonal polarization modes, are given by

$$\frac{dF_x}{dt} = -\gamma_x F_x + \kappa F_y [N_{10}A + N_{02}B],$$

$$\frac{dF_y}{dt} = -\gamma_y F_y + \kappa F_x [N_{02}A + N_{10}B],$$

$$\frac{dN_{10}}{dt} = \Gamma(N_{10} - N_1) - \kappa N_1 [F_x A + F_y B],$$

$$\frac{dN_{02}}{dt} = \Gamma(N_{02} - N_2) - \kappa N_2 [F_y A + F_x B],$$

where the coefficient $\kappa$ is proportional to the stimulated emission cross-section. $\gamma_x$, $\gamma_y$ are the cavity decay rates (inverse of photon lifetimes, $\tau_x$ and $\tau_y$, respectively) for the two modes, which we suppose to be different since the two modes experience slightly different losses. $\Gamma$ is the decay rate of population inversion (inverse of the population inversion lifetime $\tau$), which is identical for the two dipole families. $N_{10}/\tau$, $N_{02}/\tau$ are the pumping rates for the two families of atoms. Hence, $N_{01}$ and $N_{02}$ denote the corresponding unsaturated population inversions. The coefficients $A$ and $B$, which describe the strength of the interaction of each mode with each dipole orientation, are given as

$$A = \cos^2 \alpha + \beta \sin^2 \alpha,$$

$$B = \sin^2 \alpha + \beta \cos^2 \alpha.$$
It is worth mentioning that we have neglected the role of $N_3$, since it will only give terms of the order of $\beta^2$ or smaller. Thus, the coupling constant $(C)$ is given by the expression \cite{13}

$$C = \frac{4A^2B^2}{(A^2 + B^2)^2}. \quad (8)$$

Among the steady-state solution of Eqs. (2)–(5), the one corresponding to the simultaneous oscillation of the two orthogonally polarized modes is given as

$$F_x \equiv F_{x0} = \frac{\Gamma[A(r_1 - 1) - B(r_2 - 1)]}{\kappa(A^2 - B^2)}, \quad (9)$$

$$F_y \equiv F_{y0} = \frac{\Gamma[A(r_2 - 1) - B(r_1 - 1)]}{\kappa(A^2 - B^2)}, \quad (10)$$

$$N_1 \equiv N_{1h} = \frac{(A\gamma_y - B\gamma_x)}{\kappa(A^2 - B^2)}, \quad (11)$$

$$N_2 \equiv N_{2h} = \frac{(A\gamma_y - B\gamma_x)}{\kappa(A^2 - B^2)}, \quad (12)$$

where $r_1 = N_{01}/N_{1h}$ and $r_2 = N_{02}/N_{2h}$ are defined like the usual excitation ratio of a usual single-mode laser, although in this case none of them can be attributed to one mode only.

Our aim now is to calculate the fluctuations of $F_x$ and $F_y$ around the corresponding steady-state values. We start by supposing that the dominant source of intensity fluctuations for the two laser modes comes from the fluctuations of the pumping terms, which are modeled as

$$N_{01}(t) = \tilde{N}_{01} + \delta N_{01}(t). \quad (13)$$

$$N_{02}(t) = \tilde{N}_{02} + \delta N_{02}(t). \quad (14)$$

Because of the pumping fluctuations, the photon numbers related to two laser modes as well as the population inversions corresponding to the different dipole orientations will fluctuate, which we take into account as follows:

$$F_x(t) = F_{x0} + \delta F_x(t), \quad (15)$$

$$F_y(t) = F_{y0} + \delta F_y(t), \quad (16)$$

$$N_1(t) = N_{1h} + \delta N_1(t). \quad (17)$$

$$N_2(t) = N_{2h} + \delta N_2(t). \quad (18)$$

Now, substituting Eqs. (12)–(18) into Eqs. (2)–(5) and then performing Fourier transformation followed by linearization, we obtain the following expression, relating the fluctuations of the photon numbers of the two lasing modes to the pump fluctuations in the frequency domain:

$$\left[ \delta F_x(f) \right] = \left[ M_{11}(f) \right] \delta N_{01}(f), \quad (19)$$

where the tilda denotes Fourier transformed quantities and where $f$ is the frequency of the considered spectral component of the fluctuations. The coefficients of the linear response transfer matrix of the system are given by

$$M_{11}(f) = \frac{\Delta}{A(A^2 - B^2)\Gamma\left[(A\gamma_y - B\gamma_x) - \frac{2i\pi f}{\kappa\omega_0}(r_2\Gamma - 2i\pi f)\right]}, \quad (20)$$

$$M_{12}(f) = -\frac{\Delta}{B(A^2 - B^2)\Gamma\left[(A\gamma_y - B\gamma_x) + \frac{2i\pi f}{\kappa\omega_0}(r_1\Gamma - 2i\pi f)\right]}, \quad (21)$$

$$M_{21}(f) = -\frac{\Delta}{B(A^2 - B^2)\Gamma\left[(A\gamma_y - B\gamma_x) + \frac{2i\pi f}{\kappa\omega_0}(r_2\Gamma - 2i\pi f)\right]}, \quad (22)$$

$$M_{22}(f) = \frac{\Delta}{A(A^2 - B^2)\Gamma\left[(A\gamma_y - B\gamma_x) - \frac{2i\pi f}{\kappa\omega_0}(r_1\Gamma - 2i\pi f)\right]}. \quad (23)$$

The denominator $\Delta$ of these matrix elements reads

$$\Delta = A^2\left[(A\gamma_x - B\gamma_y) - \frac{2i\pi f}{\kappa F_{x0}}(r_1\Gamma - 2i\pi f)\right] \times \left[(A\gamma_y - B\gamma_x) + \frac{2i\pi f}{\kappa F_{y0}}(r_2\Gamma - 2i\pi f)\right] - B^2\left[(A\gamma_x - B\gamma_y) + \frac{2i\pi f}{\kappa F_{y0}}(r_1\Gamma - 2i\pi f)\right] \times \left[(A\gamma_y - B\gamma_x) - \frac{2i\pi f}{\kappa F_{x0}}(r_2\Gamma - 2i\pi f)\right]. \quad (24)$$

In the simple case where we consider the case of identical modes, i.e., when the laser modes have identical losses $(\gamma_x = \gamma_y = \gamma_{\text{cav}})$ and the two dipole families see equal pumps $(r_1 = r_2 = r)$, the zeros of $\Delta$ lead to the following expressions for the in-phase and antiphase relaxation oscillation frequencies:

$$f_R = \frac{1}{2\pi} \sqrt{\Gamma_{\text{cav}}(r - 1)}, \quad (25)$$

$$f_{AR} = \frac{1}{2\pi} \sqrt{\Gamma_{\text{cav}}(r - 1) \frac{(A - B)^2}{(A + B)^2}}. \quad (26)$$

Now in the experiment, the dipoles of different orientations are all pumped by the same laser, so we can take the following approximations for the pump noises corresponding to the different dipole orientations:

$$\langle |\delta \tilde{N}_{01}(f)\rangle^2 \rangle = \langle |\delta \tilde{N}_{02}(f)\rangle^2 \rangle = \langle |\delta \tilde{N}_0\rangle^2 \rangle. \quad (27)$$

$$\langle \delta \tilde{N}_{01}(f)\delta \tilde{N}_{02}^*(f) \rangle = \eta \langle |\delta \tilde{N}_0\rangle^2 \rangle e^{i\phi}. \quad (28)$$
where we take a correlation coefficient $0 < \eta < 1$, which we assume to be independent of frequency, and $\psi = 0$. So Eqs. (27) and (28), respectively, imply that the pumping fluctuations for the two modes are white noises of identical amplitudes and that they are partially correlated ($0 < \eta < 1$) with correlated parts, which are in phase ($\psi = 0$). So the two dipole orientations behave as if they were experiencing pumping fluctuations whose RIN spectra are given by the following expressions:

$$RIN_{pump1} = \frac{\langle |\delta N_0|^2 \rangle}{N_{01}^2},$$  \hspace{1cm} (29)

$$RIN_{pump2} = \frac{\langle |\delta N_0|^2 \rangle}{N_{02}^2}. \hspace{1cm} (30)$$

Similarly, the RIN spectra for the two orthogonally polarized laser modes are given as

$$RIN_{x}(f) = \frac{\langle |\delta F_{x}(f)|^2 \rangle}{F_{x0}^2}, \hspace{1cm} (31)$$

$$RIN_{y}(f) = \frac{\langle |\delta F_{y}(f)|^2 \rangle}{F_{y0}^2}. \hspace{1cm} (32)$$

Finally, the normalized spectrum of correlation between the intensity noises of the two laser modes is defined as

$$\Theta(f) = \frac{\langle |\delta F_{x}(f)|^2 \rangle}{\sqrt{\langle |\delta F_{x}(f)|^2 \rangle \langle |\delta F_{y}(f)|^2 \rangle}}. \hspace{1cm} (33)$$

This correlation spectrum $\Theta(f)$ is a complex function, with both amplitude and phase. We define the correlation amplitude spectrum as $|\Theta(f)|^2$ and the correlation phase spectrum as $\text{Arg}[\Theta(f)]$. The $|\Theta(f)| \leq 1$ condition should be satisfied, where $|\Theta(f)| = 1$ implies perfect correlation at frequency $f$.

3. EXPERIMENTAL SETUP

Figure 2(a) is a schematic of the experimental setup used to measure the correlation between the intensity fluctuations of the two orthogonally polarized oscillating modes. We have chosen a (100)-cut Nd:YAG crystal as our gain medium. The idea behind this choice of crystal cut is to assign different coupling values between the two modes by choosing different orientations of the eigenpolarization directions with respect to the crystallographic axes. More specifically, this configuration even allows us to almost completely decouple the two modes by aligning them with the (001) and (010) crystallographic axes, hence assigning almost two independent sets of dipoles to the two polarization modes as already illustrated in [13,14].

The linear cavity, closed by two mirrors, is a 9 cm long planar-concave cavity. The intracavity quarter-wave plate (QWP) determines the orientations of the two orthogonally polarized laser modes. Because the Nd:YAG active medium is optically isotropic, the two eigenpolarizations are linear and aligned along the directions $x$ and $y$ of the neutral axes of the QWP. So the eigenpolarization directions can be rotated simply by rotating the QWP and thus adjusting the angle $\alpha$ between the eigenpolarization directions and the crystallographic axes [see Fig. 2(b)]. The gain is provided by the 2 cm long (100)-cut Nd:YAG crystal, which is placed inside a copper mount to limit thermally induced birefringence. The crystallographic axes were precisely determined by x-ray diffraction. The pumping is provided by a CW multimode fiber-coupled laser diode, operating at 808 nm and delivering 300 mW of optical power. The planar cavity mirror has a maximum reflection at 1.064 $\mu$m and transmits the pump wavelength. The output coupler has a transmission of 2% and a radius of curvature of 100 mm. The dual-frequency laser is operating at 1.064 $\mu$m and its total output power is about 20 mW. The depolarized pump beam permits us to avoid any unwanted pump-induced gain anisotropy. The 1 mm thick uncoated YAG étalon located inside the cavity forces the laser to oscillate in single longitudinal mode for each polarization state. The laser spectrum is continuously analyzed with a scanning Fabry–Perot interferometer to make sure that each eigenpolarization always oscillates in single-frequency regime without any mode hopping during data acquisition. The combination of the half-wave plate and a polarization beam splitter in front of the detectors enables us to direct the intensity of two modes separately on two detectors without mixing them. We have used two identical InGaAs photodiodes (Epitaxx Inc., 3.7 MHz bandwidth), which are followed by identical low-noise homemade amplifiers. The amplified electrical

Fig. 2. (a) Schematic representation of the experimental setup. LD, laser diode; QWP, quarter-wave plate; HWP, half-wave plate; BS, beam splitter; PBS, polarization beam splitter; D, photodiode. (b) Orientations of the two eigenpolarization modes $x, y$ with respect to the crystallographic axes (010) and (001).
signals are recorded in time domain by a deep memory digital oscilloscope. They are then numerically processed to obtain the RIN and intensity noise correlation spectra.

4. EXPERIMENTAL RESULTS AND COMPARISON WITH THEORY

The RIN spectra and the noise correlation spectra have been measured for three different values of $\alpha$, namely $20^\circ$, $30^\circ$, and $52^\circ$. For $\alpha = 20^\circ$—which, according to Eq. (8), corresponds to a coupling constant $C \approx 0.09$—the RIN spectra and the noise correlation spectra are reproduced in Fig. 3. The experimental RIN spectra for the two laser modes [Fig. 3(a)] show the presence of the two relaxation oscillation peaks. The peak lying in the high-frequency region of the noise spectra ($\sim 90$ kHz) is due to the standard relaxation oscillation, whereas the low-frequency peak ($\sim 50$ kHz) is caused by antiphase relaxation oscillation [see Fig. 3(a)]. The presence of these relaxation oscillation peaks in the noise spectra confirms the class B dynamical behavior of our laser [15]. The RINs of the two laser modes, which correspond to red and blue curves in the RIN spectra [Fig. 3(a)], are not exactly identical since the intracavity losses or gains for the two modes are difficult to set perfectly identical. The corresponding theoretical RIN spectra, obtained from our model, are shown in Fig. 3(b). The value $\eta = 0.75$ taken to plot the theoretical curves has been adjusted. They show very good agreement with the experiment, except for the fact that the position of the antiphase peak is at about 65 kHz in the theoretical RIN spectra [Fig. 3(b)], instead of at $\sim 50$ kHz as shown by the experiment [Fig. 3(a)]. When one observes the experimental correlation amplitude spectrum [see Fig. 3(c)], one notices the presence of two dips at about 30 and 60 kHz, respectively. Moreover, the correlation amplitude becomes maximum (close to 0 dB) at about 50 kHz and it remains close to 0 dB for all frequencies larger than 70 kHz. The correlation amplitude is almost flat and equal to about $-5$ dB for frequencies lower than 20 kHz. The corresponding theoretical correlation amplitude spectrum is shown in Fig. 3(d). It also exhibits two dips, at about 50 and 70 kHz, whereas the correlation amplitude reaches its maximum value (0 dB) between these dips and becomes maximum again at 80 kHz and above. It is also almost flat (at $-5$ dB) for frequencies lower than 30 kHz. Hence, the theoretical correlation amplitude spectrum [Fig. 3(d)] shows nice agreement with the corresponding experimental one [Fig. 3(c)]. Finally, the experimental correlation phase spectrum [see Fig. 3(e)] exhibits two phase jumps: from 0 to $\pi$ at about 30 kHz and from $\pi$ back to 0 at about 60 kHz. The corresponding theoretical correlation phase spectrum [see Fig. 3(f)] reproduces these two phase jumps.

Fig. 3. Results for $\alpha = 20^\circ$, which corresponds to a theoretical coupling constant value $C = 0.09$. (a) Experimental and (b) theoretical RIN spectra for the two oscillating modes (red and blue plots correspond to $x$- and $y$-polarized modes, respectively), (c) experimental and (d) theoretical intensity noise correlation amplitude spectra, (e) experimental and (f) theoretical intensity noise correlation phase spectra. The following parameter values have been used for theoretical simulations: $\beta = 0.025$, $r_1 = 1.35$, $r_2 = 1.4$, $\tau_x = 1/\gamma_x \approx 5$ ns, $\tau_y = 1/\gamma_y \approx 5.2$ ns, $\tau = 1/\Gamma \approx 200$ $\mu$s, $\text{RIN}_{y,\text{pump}} = \text{RIN}_{x,\text{pump}} = -110$ dB/Hz, $\eta = 0.75$, $\psi = 0$. 

(a) RIN (dB/Hz) vs. Frequency (kHz)
(b) RIN (dB/Hz) vs. Frequency (kHz)
(c) Correlation Amplitude (dB) vs. Frequency (kHz)
(d) Correlation Amplitude (dB) vs. Frequency (kHz)
(e) Correlation Phase (degree) vs. Frequency (kHz)
(f) Correlation Phase (degree) vs. Frequency (kHz)
The theoretical plots of Fig. 3 have been obtained with the approximations of white pump noise, i.e., independent of frequency in the considered frequency range (0–120 kHz). We have taken a pump RIN value of −110 dB/Hz. This value was determined experimentally by measuring the pump RIN, as shown in Fig. 4. This pump RIN has been measured after a polarizer and it was found that the pump RIN is white and does not depend on the orientation of the polarizer. Moreover, its value is equal to −110 dB/Hz, which is exactly the value we took for the theoretical simulations throughout this paper.

Figure 5 reproduces the results obtained for α = 30°, which corresponds to a coupling constant C equal to 0.4, according to Eq. (3). The experimental and theoretical RIN spectra for the two laser modes are shown in Figs. 5(a) and 5(b), respectively. In the experimental RIN spectra [Fig. 5(a)], we find the in-phase peak at ∼80 kHz and antiphase peak at ∼50 kHz. The RIN spectra for the two modes are again slightly different, since the intracavity losses and gains are different. The corresponding theoretical RIN spectra, as shown in Fig. 5(b), show good agreement with the experiment apart from the fact that the antiphase peak is at about 40 kHz [Fig. 5(b)], although experimentally we find it at about 50 kHz [Fig. 5(a)]. The experimentally measured noise correlation amplitude and phase spectra are shown in Figs. 5(c) and 5(e), respectively. There are two dips in the correlation amplitude spectrum at about 25 and 55 kHz, as shown in Fig. 5(c). Moreover, in Fig. 5(c), we also see that the correlation amplitude reaches 0 dB once at about 50 kHz, and again at 70 kHz, and maintains 0 dB value for all higher frequencies. Now in the experimental correlation phase spectrum [Fig. 5(e)], we find two jumps, one from 0 to π at about 25 kHz and the other one at about 55 kHz from π to 0. Moreover, the correlation phase is 0 for frequencies between 0 and 25 kHz and also for frequencies higher than 55 kHz, whereas it is π within 25–55 kHz. The theoretical correlation amplitude and phase spectra as reported in Figs. 5(d) and 5(f), respectively, show satisfactory agreement with the experiment [Figs. 5(c) and 5(e), respectively].

Finally, the results for α = 52°, which correspond to a relatively high coupling constant (C ≈ 0.75) according to Eq. (3), are shown in Fig. 6. Now the in-phase relaxation oscillation peak lies at 100 kHz as exhibited by both experimental results [Fig. 6(a)] and theoretical results [Fig. 6(b)]. The positions of the antiphase peak are slightly different as obtained from the experiment [at 60 kHz as in Fig. 6(a)] and theory [at 30 kHz as in Fig. 6(b)]. The RINs for the two modes, which are denoted by the red and blue curves in Figs. 6(a) and 6(b), are slightly different because of the difference in intracavity losses or gains for the two modes. Figures 6(c) and 6(d) represent, respectively, the experimental and theoretical correlation amplitude spectra. In the experimental correlation amplitude spectrum [Fig. 6(c)], we now find only one dip, located at about 70 kHz. The corresponding theoretical spectrum [Fig. 6(d)] confirms the fact that there is only one dip left, even if its frequency is slightly different (40 kHz). Moreover, the experiment shows that [Fig. 6(c)], starting from ∼−10 dB at about 0 kHz, the correlation amplitude gradually reaches maximum value (0 dB) at about 60 kHz and it maintains maximum value (∼0 dB) for all frequencies higher than 90 kHz. The theory also predicts the same kind of behavior of the correlation amplitude spectrum [Fig. 6(d)], except for the fact that, theoretically, correlation amplitude reaches 0 dB at frequencies 30 and 60 kHz rather than at frequencies 60 and 90 kHz, respectively, as shown by the experiment [Fig. 6(c)]. Now, as shown in Fig. 6(e), the experimental correlation phase is π for frequencies 0–70 kHz, and then it jumps from π to 0 at about 70 kHz and remains at 0 for all the higher frequencies. The corresponding theoretical correlation phase spectrum, as shown in Fig. 6(f), shows satisfactory agreement with the experiment [Fig. 6(e)], except for the fact that the phase jump from π to 0 occurs at about 40 kHz and not at about 70 kHz as shown by the experiment [Fig. 6(e)].

For all the situations described previously corresponding to different α values and hence to different coupling strengths between the two lasing modes, the values of γ1 and γ2 taken for the theoretical simulations have been estimated from the experiment by measuring the ratios of the pump power to the threshold pump power. The values of γr1 and γr2 have been adjusted, knowing the cavity length (9 cm) and the transmission loss of the output mirror of the cavity (∼2%). The value of τ (200 μs) has been used from the knowledge of the preceding experiments performed with this active medium [14].

5. PHYSICAL INTERPRETATION OF THE RESULTS

Our results show that the RIN spectra and the intensity noise correlation spectra are dependent on the angle α between the crystallographic axes and the eigenpolarizations, which determines the strength of coupling between the two laser modes. In the following, we try to give a physical insight to these results.

To begin with, one can draw the analogy between a single-frequency laser and a simple mechanical oscillator such as a mass attached to a spring. If this oscillator is driven out of its steady state, it will relax to its steady-state position either exponentially or in damped-oscillatory manner depending on the strength of damping. In the same way, the relaxation of a single-frequency laser to its steady-state intensity can be either exponential or damped-oscillatory, depending on whether the laser dynamics are of the class A or class B type, respectively. Therefore, the two-frequency laser in which the two orthogonally polarized modes are coupled can be thought of as very analogous to a two-coupled mechanical oscillator system. As we know for the two-coupled mechanical oscillator system, if the two oscillators are identical, then the relaxation mechanism (either exponential or damped-oscillatory) of the whole system can be generalized by two
eigenrelaxation mechanisms of the whole system: (i) in-phase relaxation mechanism and (ii) antiphase relaxation mechanism \[9\]. These two relaxation mechanisms exhibit different eigenfrequencies and/or damping rates. Moreover, for our dual-frequency Nd:YAG laser, the relaxation mechanisms are of the damped-oscillatory type because of its class B dynamical behavior. Furthermore, as the two laser modes are coupled in the dual-frequency Nd:YAG laser, there are two relaxation oscillation mechanisms, namely in-phase and antiphase relaxation oscillations, quite analogous to the two-coupled mechanical oscillator system. The transfer functions of the in-phase and antiphase response are calculated by diagonalizing the \(2 \times 2\) matrix of Eq. \(19\). To this aim, let us define the in-phase and antiphase intensity fluctuations:

\[
\delta \tilde{F}_{\text{in}} = \frac{1}{2} (\delta \tilde{F}_x + \delta \tilde{F}_y),
\]

\[
\delta \tilde{F}_{\text{anti}} = \frac{1}{2} (\delta \tilde{F}_x - \delta \tilde{F}_y).
\]

For the sake of simplicity, let us suppose that the two modes have equal parameters, i.e., that the two laser modes have identical losses \(r_x = r_y = r_{\text{cav}}\) and the two dipole families see equal pumps \(r_1 = r_2 = r\). Then we derive the normalized transfer functions for the in-phase and antiphase response as

\[
T_{\text{in}} = \frac{\left| \delta \tilde{F}_{\text{in}} \right|^2}{\left| \delta N_0 \right|^2} = \frac{2\Gamma^4 (r-1)^2 (1 + \eta)}{\Gamma r_{\text{cav}} (r-1) - 4\pi^2 f^2} + (2\pi f \Gamma)^2,
\]

\[
T_{\text{anti}} = \frac{\left| \delta \tilde{F}_{\text{anti}} \right|^2}{\left| \delta N_0 \right|^2} = \frac{2\Gamma^4 (r-1)^2 (1 - \eta) \left( \frac{r_x}{r} + \frac{r_y}{r} \right)^2}{\Gamma r_{\text{cav}} (r-1) - 4\pi^2 f^2 + (2\pi f \Gamma)^2}.
\]

These transfer functions are shown in Fig. 7 for three different coupling situations. One can notice, as shown by Eq. \(37\), that the antiphase peak should disappear for \(\eta = 1\), i.e., for perfect correlation of the pump noises seen by the two dipole families, whereas in-phase fluctuations are always present. The presence of the antiphase peak in the experimental RIN spectra for all the coupling situations presented in the paper signifies that the pump fluctuations as seen by the two dipole families are not perfectly correlated, i.e., \(\eta < 1\). Moreover, from these transfer functions, one can understand the noise response of
any one of the lasing modes. Indeed, since this response is a superposition of the in-phase and antiphase responses, it can be dominated by either one or the other of these two mechanisms.

Let us start with the small coupling case \( C = \frac{0.09}{0.0136} \), as shown in Fig. 7(a). One can see that the in-phase response dominates over the antiphase one in the 0–50 kHz frequency range. Then, for frequencies between 50 and 70 kHz, the antiphase response becomes dominant over the in-phase response. Finally, the in-phase response becomes stronger again for all the frequencies higher than 70 kHz. This explains why, for \( \alpha = 20^\circ \) [see Figs. 3(e) and 3(f)], the phase of the correlation jumps from 0 at low frequencies, to \( \pi \) at intermediate frequencies, and then back to 0 at high frequencies. These phase jumps correspond to crossings between the responses of the two relaxation mechanisms in Fig. 7(a). They also correspond to the frequencies for which the intensity noise correlation amplitude exhibits a minimum [see

![Normalized transfer functions for the in-phase (red line) and antiphase (black line) relaxation oscillation mechanisms for three different values of \( \alpha \), corresponding to three different coupling situations: (a) \( \alpha = 20^\circ \), (b) \( \alpha = 30^\circ \), and (c) \( \alpha = 52^\circ \). The values of the other parameters used for the simulations are \( \beta = 0.025 \), \( r_1 = r_2 = 1.4 \), \( r_e = r_g = 5 \) ns, \( \tau = 200 \) µs, \( \eta = 0.75 \), \( \psi = 0 \).](image)
Figs. 3(c) and 3(d)]. Moreover, the fact that the correlation amplitude is rather low (≤−5 dB) for frequencies lower than 20 kHz, as shown in Fig. 3(d), comes from the facts that (i) the pump noises are partially uncorrelated (η = 0.75 < 1), and (ii) the transfer functions of the two mechanisms are separated by only one order of magnitude in this region [see Fig. 7(a)], leading to a partial cancellation of the noise. By contrast, in the high-frequency region (above 80 kHz), the two transfer functions are separated by several orders of magnitude and the correlation amplitude becomes very close to 1.

The interpretation is quite similar in the case of the moderate coupling case C = 0.40 [see Fig. 7(b)], corresponding to the experimental results obtained for α = 30° (see Fig. 5). The main differences with the preceding case are that (i) the antiphase relaxation oscillation frequency is smaller, leading to the fact that the two transfer functions of Fig. 7(b) cross at lower frequencies, which is consistent with the fact that the phase jumps for the correlation occur at lower frequencies in Figs. 5(e) and 5(f) compared with Figs. 3(e) and 3(f); (ii) the difference between the transfer functions of the two mechanisms becomes very small in the low-frequency region (below 20 kHz), explaining why the correlation amplitude becomes even smaller in the low-frequency region [see Figs. 5(c) and 5(d)].

Finally, for the relatively stronger coupling case C = 0.75 [see Fig. 7(c)], corresponding to the experimental results obtained for α = 52°, one can see from Fig. 7(c) that the antiphase relaxation oscillation frequency has decreased so much that the first crossing of the two transfer functions has disappeared: the antiphase mechanism dominates at low frequencies, and the in-phase mechanism dominates at high frequencies. This explains why there is only one phase jump in the spectra of Figs. 5(e) and 5(f), and only one dip in those of Figs. 5(c) and 5(d).

It is worth noticing that the possible existence of two phase jumps, associated with two intersections of the transfer functions, is due to the fact that these transfer functions are resonant, which is linked with the class B dynamical behavior of our Nd:YAG laser. This strongly contrasts with the case of the class A lasers that we recently discussed in the case of a dual-frequency VECSEL [12]. Indeed, for a class A laser, the transfer functions of the relaxation mechanisms look like the transfer functions of a first-order filter or of an overdamped oscillator and decrease monotonically with frequency. This implies that they can exhibit only zero or one intersection, leading to the fact that a maximum of one phase jump (and sometimes no phase jumps) can be observed in the correlation spectrum [12].

Further, one can notice by comparing the theoretical and experimental spectra in Figs. 3, 5, and 6 that the shift of the antiphase relaxation oscillation frequency when α is changed is not as large in the experiment as expected from theory. This may be due to fact that the coupling strength might not exactly vary with α as predicted by our model. One possible explanation of this discrepancy may lie in the fact that our model supposes the existence of only three families of dipoles oriented along the crystallographic axes of the gain medium [13], although several papers suggest more complex descriptions for the spectroscopy of Nd³⁺ ions embedded in YAG matrix [16–19]. Moreover, our model does not take into account the effect of spatial hole burning [20,21], which must lead to a reduction of C which could partially explain why C does not evolve in such a large range.

6. CONCLUSION

The behavior of the RIN spectra as well as the correlation between the intensity noises of the two orthogonally polarized modes of a dual-frequency Nd:YAG laser have been investigated both experimentally and theoretically. Moreover, the dependence of the RIN spectra as well as the intensity noise correlation spectra on the strength of coupling has been explored. Since the dual-frequency Nd:YAG laser is class B type, this intensity noise correlation study generalizes the same type of noise correlation study done with dual-frequency VECSEL [12], which was a class A laser. The presence of the relaxation oscillations for the dual-frequency Nd:YAG laser greatly modifies the behavior not only of the RIN spectra, but also of the noise correlation spectra with respect to the class A laser. For example, although the pump noises, which are the only source for the laser modes, are only partially correlated, the intensity noises for the two laser modes become fully correlated (0 dB exactly) for frequencies around the relaxation oscillation frequencies. The theoretical model, based on the assumption that the dipoles inside the gain medium are aligned along the crystallographic axes of the active medium [13], shows fairly good agreement with the experiment. The remaining discrepancy regarding the shift of the antiphase peak with the change of α gives the opportunity for further developments of the model to take into account other effects (for example, spatial hole burning), which could modify the coupling strength.

Of course, it would be very interesting to extend the present work to other laser materials, such as YAG doped with other ions, or other crystal matrices, or even doped glasses. This implies a careful study of the spectroscopy of these materials, which is not always available in the literature, in order to understand the orientation of the absorbing and emitting dipoles.

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